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THE USE OF GREGARINES IN THE LABORATORY AS TYPICAL OF A PROTOZOAN CLASS.

BY MINNIE E. WATSON,

Oyster Bay, N. Y.

Teachers may find some of the *Gregarines* valuable as a substitute for the *Paramœcium* or *Amœba,* or as a supplementary study, illustrating the Sporozoa. The parasites may be studied to illustrate form, movement and method of procuring food, and, possibly, method of reproduction, all unique to the Sporozoa.

DISTRIBUTION.

Gregarines are very easily procured. They live as parasites in the alimentary tract of various arthropods. In the Middle West, the common red-legged grasshopper, *Melanoplus femurrubrum* and its allies (*M. differentialis, M. bivitattus, Encoptolophus sordidis,* etc.) harbor the large biassociative *Gregarina rigida* (Hall) Crawley. In some localities, eighty-five per cent of the grasshoppers are parasitized in the fall months and the infection is very heavy in each host. The green field grasshoppers which are plentiful in the spring will be found to contain a few parasites and the same may be said of the small brown nymphs of the previously mentioned fall species, which are available in the spring. Crickets sometimes contain Gregarines, but the infection is slight.

In the East, the grasshoppers seem to be uninfected but the common field cricket, *Gryllus pennsylvanicus,* contains several species of Gregarines, the one most frequently met with being the solitary *Leidyana solitaria* Watson. In some of the eastern states about seventy-five per cent of the crickets examined were found to be infected.

Various beetles—Lucanidae, Dytiscidae, Gyrinidae, Carabidae, etc., as well as other Orthoptera than those mentioned—are sometimes infected in both the East and Middle West but the infection is sporadic and therefore unreliable for class purposes.

METHOD OF OBTAINING PARASITES.

Clip off both ends of the body of the host between the mouth and anus and with forceps withdraw the alimentary tract intact. Place it on a slide with a drop of normal salt solution (or water) and with fine scissors slit the tube lengthwise and spread it out flat by carefully teasing the walls apart with two needles set in handles. Remove any solid food masses. Lay the cover slip on the tissue carefully so as not to crush the fragile Gregarines. The parasites of the grasshopper may be seen with the naked eye as minute, whitish bodies sometimes clustered in rosettes. Those of the cricket are not visible to the unaided eye. After a few minutes the parasites, if present, will begin to move outward from definite foci toward the periphery of the cover slip and can then be studied. Normal salt solution facilitates their movement and enables them to remain alive longer than if water is used. The motion of Gregarines is slow and generally progressive with or without a gentle bending of the body and the animals can easily be kept in the field.

The parasites disintegrate rather rapidly in artificial media and the hosts should not be opened until the student is ready to examine them for parasites. Each student should be supplied with several host-insects so that at least one will be found parasitized.

For detailed descriptions of Gregarines, see Doflein, *Lehrbuch der Protistenkunde;* Delage et Herouard, *Traite de Zoologie Concrète,* Vol. 1; Minchin, in *Lankester's Treatise on Zoology,* Part I, Fac. 2; Minchin, *An Introduction to the Study of Protozoa.*

Gregarines save time for the busy teacher. Instead of procuring and growing cultures of the free-living protozoa for weeks, simply collect a few grasshoppers or crickets and examine them for parasites. The hosts may be kept for a few days in the laboratory before being used. If crickets are fed moistened cracker crumbs, the intestinal contents will be white and readily distinguishable from the darker parasites.

If no syllabus is being followed which requires the study of Paramœcium, the Gregarines form an excellent substitute because (1) they are visible with a low power of the microscope (x 120 being sufficient for most purposes); (2) they are easily distinguished by the student because of their density and brown or black color; (3) are easily kept in the field because of their slow uniform movement; (4) and form probably the only parasitic type with which the student can easily become acquainted in the laboratory.

A LABORATORY EXERCISE ON GREGARINES.

Morphology.

After a parasite is found and located on the slide (three ocular, three objective), note whether it is attached to another individual, i. e., is associative, or solitary. How many individuals are present in the association (if associative)? What is each called? How are the individuals attached together, laterally or by their poles? Into how many parts is each divided? What is the comparative length of the two parts? What is the name of each part? (See Figure 1.) General shape of the body? Shape of the protomerite and of the deutomerite? Are protomerite and deutomerite completely separated from each other, or not?[1] What is the color of the endocyte? Is the protoplasm dense or sparse? Is it homogeneous? Do you find a nucleus? If so, where? Its shape? Content? (The small bodies sometimes visible within the nucleus are karyosomes—nodules of chromatin material.)

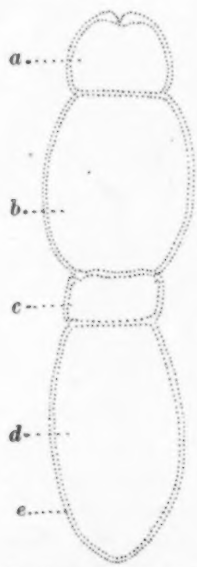


FIG. 1. Outline of an association of two mature sporonts of *Gregarina rigida* (Hall) Crawley. (a) Protomerite of first individual (the primite). (b) Deutomerite of primite. (c) Protomerite of satellite. (d) Deutomerite of satellite. (e) The epicyte, or outer layer of the cell.

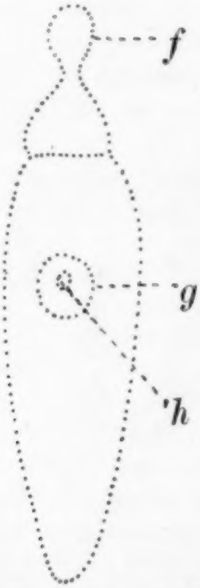


FIG. 2. An immature individual of the solitary species *Leidyana solitaria* Watson with a holdfast. (f) The epimerite. (g) The nucleus. (h) Karyosome within the nucleus.

[1] The thin layer between them is the sarcocyte, continuous with the same layer surrounding the body.

Movement.

Watch an individual which is in motion for a few seconds. (If none is moving, add slowly, with a pipette, a drop of normal salt solution. If this does not stimulate movement within a few seconds, examine a fresh host, for the parasite may be dead, especially if it has been on the slide half an hour or more). Is motion uniform or irregular? Write a sentence about your decision, stating why you decide thus. Do you detect any organs for movement? Are there any cilia? (This question if the student has previously studied Paramœcium.) Is any part of the body extruded or retracted, as is often the case with the Amœba?

Food.

Do you note a mouth? Is there any visible organ for excretion? Have you seen the animal move toward possible food masses in the debris?

How do internal parasites in general obtain their food, e. g., the tapeworm? (Look this up in your text.) The Gregarine obtains its nourishment by absorption of chyle and other liquids from the host intestine through its body walls which are very porous. Excretion takes place in the reverse direction in the same manner. The Gregarine therefore has no need of specialized mouth and anus.

Reproduction.

Have you noticed in the intestinal matter from the host intestine any dense white spherules about the size of an association of two Gregarines? If so, these are the cysts, formed from the union of two individuals, which revolve together in an ever narrower spiral direction, form a compact ball which rotates for an hour or more after it is formed and finally comes to rest. The outer layer of this cyst is the epicyst.

After two or three days in a suitable, moist medium, the cyst will have developed from two to fifteen spore-ducts, leading outward from the cyst and through these will emerge, in chains, the ripe oval or barrel-shaped spores. It is difficult and well-nigh impossible to procure the development of cysts of most Gregarines in artificial media and is scarcely advisable for the student to attempt.

In the host, the cyst is dropped with the feces and further development occurs outdoors. The spores are scattered broadcast and when eaten with its food by an insect of the same spe-

cies as the host, will develop in the intestine, the outer wall being digested off. Each spore develops into (generally eight) tiny falciform sporozoites and the latter move in an ameboid manner to and then into the epithelial cells of the intestine where they either completely or partially embed themselves. Those which become partially embedded develop holdfasts called epimerites.

When the young parasites have "attained their majority," they emerge from the host cell (which has been mostly or entirely digested by the parasite) or lose their epimerites. Henceforth they lead a free existence in the intestine of the host and begin over again the cycle of reproduction.

A MODERN VIEW OF VALENCE.

By Robt. W. Boreman,
High School, Parkersburg, W. Va.

One of the subjects that seems hard for the average elementary student to grasp is valence. Possibly an article on the subject will be timely if printed about the time valence is discussed in most of our preparatory and high schools.

The following are a few of the definitions given in some of the recently published texts on chemistry:

"The valence of an element is that property which determines the number of the atoms of another element which it can hold in combination."

"Valence is the power that an atom has of combining with other atoms to form molecules."

"The number which expresses the combining power of an atom of an element is called the valence of the elements."

These texts all discuss valence more or less at length without once mentioning the present-day view of valence. This view should not be beyond the ability of the preparatory school student to understand.

Modern physicists consider that the ultimate composition of matter is negative electricity made up of minute particles or atoms of pure electricity called electrons. It should be noted that the electron is pure electricity and not electricity seated on a particle of matter. Poincare had the following conception of the atom about three years ago:

"Each atom is like a kind of solar system where the small negative electrons play the rôle of planets revolving around the

great positive central electron which takes the place of our sun * * * Besides these captive electrons there are others which are free and subject to the ordinary kinetic laws of gases. The second class are like the comets which circulate from one stellar system to another, establishing thus an exchange of energy between distant systems."

This comes close to being the modern view, except that the central nucleus is minutely small. The electron has no mass but has what corresponds to mass, due to its charge which gives it a certain electrical inertia. However, we popularly use the word "mass" in speaking of an electron, and we say it has about one-thousandth the mass of an atom of hydrogen. In a given atom there is no excess of either positive or negative charge. Each atom of a given element contains a certain definite number of electrons, but the number differs in the different elements, increasing with their atomic weights.

Now, it so happens that some of our elements tend to lose one electron very easily, thus leaving the atom positively charged, as is the sodium atom. Other atoms on the contrary have an affinity for electrons and tend to take on one or more, making the atom as a whole electronegative. Example, chlorine.

In a compound, therefore, like sodium chloride, we find the sodium tending to give off an electron and the chlorine tending to acquire an electron, and the natural conclusion is that both tendencies will be satisfied, the sodium being left positively charged and the chlorine negatively charged, thus, Na^+, Cl^-.

It is quite likely, therefore, that the atoms in the molecule are held together by the attraction of one charge of electricity for another of different sign, also some of the metallic elements tend to give off one electron, leaving the element with part of its positive charge unneutralized, such as potassium which is univalent; some tend to give off two electrons, leaving the atom with two unneutralized charges of positive electricity as zinc which is bivalent; some tend to give off three electrons, leaving the atom with three unneutralized charges of positive electricity as aluminium which is trivalent, and so forth. The same is true for the elements that take up these electrons, thus themselves becoming negatively charged.

Considering these facts we may safely define the valence of an element as being the *capacity* the atoms of that element have of holding electric charges—positive or negative.

THE ANALYSIS OF SOME WELL-KNOWN ROCKS.

By Nicholas Knight,
Cornell College.

1. The Rock of Which the Houses of Parliament Are Constructed. The specimen was sent us some months ago by the Right Honorable John Burns, member of the British Cabinet. We read in one of the popular magazines that the rock in the Parliament buildings is quite rapidly weathering and crumbling, which indeed a close inspection of the building readily confirms. We desired to make the chemical analysis to ascertain, if possible, the reason for the crumbling. The rock is a beautiful buff color when first quarried, and on a fresh fracture, but it is blackened by the London smoke, except in some of the more protected angles. The smoke is so thick on the rock that it can be easily rubbed off by a piece of paper or a handerchief, which we tested in many places throughout the building. The analysis was made by Miss Bonnybel Artis, and the figures obtained are as follows:

	Per cent.
$Si\ O_2$	1.85
$Fe_2\ O_3$	0.60
$Al_2\ O_3$	1.54
$Ca\ CO_3$	46.32
$Mg\ CO_3$	49.70
Total	100.01

The specific gravity is 2.60.

J. Allen Howe, curator of the Jermyn Street Museum, London, writes:

"At the request of Sir Archibald Geikie, I have pleasure in sending you the following short note on 'Anston Stone,' used in the Houses of Parliament.

"Auston Stone is, as you say, a dolomite and light buff in color on the fresh fractured surface. It still retains traces of this tint in some of the inner quadrangles of the building, but where more exposed it has become a dark gray. I am quite unable to explain why it should have appeared to you to be red, unless it was seen about sunset.

"The stone comes from quarries at Kiveton Park, east of Sheffield in Yorkshire, where it occurs in the magnesian limestone division of the Permian formation. Other quarries in the same kind of stone are worked at Steetley and Mansfield (red and white). Very little sandstone is used by London builders."

The stone is used in many of the public buildings of London, such as the Bank of England, the Mansion House, Westminster Abbey, St. Paul's Cathedral, and also in many private houses. Neither the chemical composition nor the physical characteristics explain the weathering of the rock. In appearance it seems not unlike, or at least not widely different from, the Roman travertine, which is still standing in buildings erected 2,500 years ago. The Houses of Parliament were constructed in 1846-62. We desire to record our thanks to Honorable John Burns for so kindly taking the trouble to send us a large specimen of the rock.

2. Red Sandstone from the Vosges Mountains.

This is a handsome, durable rock from the Vosges Mountains, of which the Strassburg Cathedral and other public buildings in the city are constructed. The most interesting of all the buildings is the cathedral. The first church was erected in 600 A. D., and the present structure in the eleventh and twelfth centuries. It is a mixture of Romanesque and Gothic.

The most beautiful portion is the facade, which is also the purest Gothic, a form of architecture that originated in northern France. The numerous statues of prophets, sibyls and apostles and the fine carvings in the stone, like crytallized lace, show how the material is adapted to the most beautifully artistic stone work. The tower is nearly 500 feet in height, and therefore one of the tallest structures in Europe. It is likewise Gothic, and exquisitely wrought, although in many places hit by the shells of the Prussians in the War of '70-'72. The rock is easily transported from the quarries in the mountains on the River Ill, which bisects the city of Strassburg.

The analysis was made by Ellery Botts, and resulted as follows:

	Per cent.
$Si\ O_2$	77.57
$Al_2\ O_3$	14.45
$Fe_2\ O_3$	4.50
$Ca\ O$	1.62
$Mg\ O$	0.80
$K_2\ O$	0.21
$Na_2\ O$	0.45
$H_2\ O$	0.45
Mu O	0.00
$C\ O_2$	0.00
$Ti\ O_2$	0.00
Total	100.05

America, doubtless, has as fine and durable building stones as any quarter of the globe, but the high cost of labor has prohibited their use to the fullest extent. As wood has advanced greatly in price in recent years, this may result in a wider use of American building stones.

EARTH RESISTIVITY.

Oil in sand or earth causes it to have a very high resistance to the flow of an electrical current, that is, speaking technically, to have a very high resistivity. Certain valuable ores in the earth cause it to have a very low resistivity. For any particular specimen of earth the resistivity varies with the moisture content. The damage to pipe systems on account of electrolysis by the return current of street railway systems depends among other things upon the resistivity of the earth around the pipes and near the tracks. There are therefore many reasons why we may wish to know the resistivity of certain very limited portions of the earth.

In a recent publication of the Bureau of Standards, Department of Commerce, a method for measuring earth resistivity which is free from some of the faults of methods which have previously been used is described. The method is particularly adapted to those cases in which it is important that the measurement be made without disturbing the earth, as is necessary where a sample is taken into the laboratory for measurement, and in those cases where we wish to measure resistivity of a fairly large portion of earth, extending to a considerable depth.

To those interested, a copy of the paper, *Scientific Paper, No. 258,* will be sent on request addressed to the Director, Bureau of Standards, Washington, D. C.

HOW THE LINE-WIRES LIGHT LAMPS—A BEGINNER'S LESSON IN ELECTRICITY.[1]

BY F. F. GOOD,
Columbia University.

An effective scheme for introducing a practical study of electricity which we have employed for a number of years begins with the flow of the current in the line-wires. Two heavily insulated wires fifteen feet long are suspended at a convenient height above the lecture table with a throw-switch connection to the 110-volt current. At intervals of about fifteen inches insulation is removed to provide for making convenient contacts. These connections furnish a means of making direct connections on the line-wires at ten or twelve different points. There is very little danger of short cricuiting the wires so long as they are kept reasonably tight. For the average lecture table the wires may be suspended from two uprights of wood, $1\frac{1}{2} \times 2\frac{1}{2}$ inches. They should be fastened against the ends of the table in such a way that they may be easily removed when not in use. For purposes of experimentation and as a safety device, two strong binding posts may be placed in the circuit for attaching pieces of 10, 15 or 20 ampere fuse-wire.

Here we have a dozen contact points as sources of current along the wires. They are in plain view of the class with provision for making quick attachments for a great variety of demonstrations and experiments. A voltmeter, an ammeter and a wattmeter may be connected to the wires at one end of the table. They should be movable and of large size with visible connections.

At the edge of the lecture table runs a gas pipe with gas cocks at intervals along the line. These gas cocks will represent on the gas line what the bared spaces do on the electric wires overhead. We may let the gas escape from one or more of these cocks and light them turning some on with a very small flame and others with the cock full open. This gas pipe ends at one end of the table. The other end passes through the floor to the basement and makes connection with the street gas mains. From there it leads back to one of the large city gas tanks. Gas is forced into the pipes by the weight of the heavy steel tank. The amount of gas flowing into our building depends of course upon how many gas cocks are opened and how wide they are opened. If we

[1]Part of a Lecture-demonstration before the New York Physics Club.

should break the end of the large gas pipe a great volume would be allowed to ascape.

Now when we consider how the gas flows we have a convenient and simple key for explaining how the electric current flows over the wires. In many important respects the electric current flows on the wires like the gas flows in the pipes. It is most convenient to consider that the electric current comes from the power house on the wire nearest to me and returns on the other wire. An examination of these wires will show that the two ends do not touch. The electric current cannot pass off the wire into the air. It must have some conductor to travel upon. Air, like glass, porcelain and rubber, is a very poor conductor. Copper is one of the best substances for carrying the electric current—a good conductor. Silver is even better than copper but it is too expensive. The return wire is like the drain pipe from the water faucet. The current under strong pressure is waiting its chance to get across to the second wire and return to the power house. The current has a chance to flow only when we place a conductor across the two wires—a kind of bridge.

I touch one wire with my thumb and the other with my first finger and note a slight tingling as a very little current passes through them for my fingers are not good conductors. If I dampen the ends of my fingers the contact is made more perfectly. When I again touch the wires more current goes through and I feel a sharp sting. The muscles tend to contract and pull away. Let us try a better conductor. Here is a 16-candle power carbon lamp with two copper wires soldered to it with ends bent to serve as hooks. When I hang it on the wires it lets enough current through to cause the filament to become glowing hot. Thus a lamp on the wires serves the same purpose as opening a gas cock on the gas pipe. By opening the gas cocks we let some of the gas escape from the pipe. By connecting a lamp across we let some of the electric current escape from one wire through the lamp to the other. Hanging more lamps on the wires at other places lets more current across. The ammeter at the end of the table indicates how much current is permitted to pass over. Hundreds of amperes are waiting for a chance to break across but each lamp lets only a small amount through. When we hang a very large lamp on it is like opening the gas cock very wide and letting a large flow pass through. If no lamps are on the wires no current can flow.

Even a lamp is not one of the best conductors. Let us see what happens when we touch the two wires with a small iron wire. A great splash of fire! The piece of iron wire bursts into a thousand sparks and they dance over the lecture table. For a moment a great lot of current leaped across on the good conductor, iron, and heated it beyond its melting point. It fell upon the table. This experiment caused the sudden bursting of our electric pipe but it was only for a moment. As soon as the wire was burned the path was again broken. This shows why the electric wires are very dangerous and should always be handled by some one who understands them. Buildings are sometimes burned because the wires accidentally touch each other at places where they should be well insulated. In buildings all wires should be well covered like these line-wires with some good insulating material.

On the voltmeter at the end of the table we may see how many volts of pressure this current has in its effort to burst over to the other wire. The voltmeter indicates 110 volts of pressure. Most house-currents have a pressure of 110 volts. The pressure on these wires always remains about 110 volts and for this reason is not very dangerous to touch with the fingers. The current which runs the street cars has about 550 volts pressure and is much more dangerous. If I should touch my fingers to the 550-volts current I might receive a bad burn. The high pressure currents must be handled with the greatest care. The pressure used in Sing Sing prison to electrocute murderers is about 2,000 volts. Some may imagine this to be a cruel death but 2,000-volts pressure instantly strikes a man unconscious, so the unfortunate fellow never knows anything about it.

On the lecture table you will see a great many lamps of different sizes, each having wires attached for hanging across the line-wires. The larger the lamp, the more current it lets through. The size of the lamp is marked on a small label. Here is the Midget Mazda lamp. If you watch the ammeter when I hang it over the wires you can see how much current it takes—about one-tenth of an ampere. The Midget Mazda gives as much light as eight candles so it is called an eight-candle power lamp. The filaments of Mazda lamps are made of the rare metal tungsten which becomes heated brilliantly white when enough current passes through it.

How much does it cost to burn the Midget for one hour?

In order to find the cost we must know how many watts are being used. If we know the volts pressure, 110, and how many amperes the lamp lets through we can easily find the watts.

Volts x Amperes = Watts.

The Midget requires 110 volts and it takes one-tenth of an ampere.

110 volts x .1 ampere = 11 watts.

Now the electric company charges 10 cents for a 1,000-watt current for each hour—10 cents per kilowatt hour. At this rate one cent would pay for a 100-watt current for an hour. The Midget Mazda takes about 10 watts and would therefore cost about one-tenth of a cent to operate it one hour. It could be run ten hours for one cent.

Our next lamp is twice as large as the Midget. It takes twice as much current, .2 of an ampere, to heat its filament. It is a 16-candle power lamp, and so we have other lamps 32 C. P., 48 C. P., 96 C. P., 150 C. P., 250 C. P., and 500 C. P. We can find how much it costs to operate these in the same way that we did the Midget if we multiply the volts by the amperes and find how many watts each lamp uses.

8 candle power	.........110 volts	x .1 ampere	= 11	watts
16 candle power	.........110 volts	x .2 ampere	= 22	watts
32 candle power	.........110 volts	x .4 ampere	= 44	watts
48 candle power	.........110 volts	x .5 ampere	= 55	watts
96 candle power	.........110 volts	x 1. ampere	= 110	watts
150 candle power	.........110 volts	x 1.5 amperes	= 165	watts
250 candle power	.........110 volts	x 2.5 amperes	= 275	watts
500 candle power	.........110 volts	x 5. amperes	= 550	watts

If it costs one cent per hour for a 100-watts current, what will be the cost of each lamp in this series?

The old style of lamps have carbon filaments and they do not give as much light as the tungsten filaments. The tungsten filament lamps are more expensive at first cost but they give more than two times as much light for the same amount of current used. Let us wrap this 60-watt Mazda lamp in a cloth and crush it with a board in order to examine the tungsten filaments. When a bulb is broken the loud report is caused by the sudden inrush of air as the glass breaks, for the inside is a vacuum. A filament would soon burn up if the air is not taken out of the bulb. Some lamps are now filled with nitrogen gas to prevent the filament from burning. We might perform an experiment to

see how long the tungsten lamp will burn if the tip of the bulb is snapped off and the air allowed to enter. With air inside, the filament breaks almost as soon as it touches the electric wires and the bulb fills with tungsten smoke.

The tungsten filament in this 60-watt lamp is about two feet long. My 11-year-old friend, Jack, dropped in the other day when I was experimenting with some lamps. He measured the thickness of the filament with a micrometer screw and concluded that it was about half as thick as a hair and one-fourth as thick as a No. 50 cotton thread. It looks much thicker than that when it is heated on the line-wires. Tungsten metal is better than other metals for making lamp filaments because it will withstand so much heat without melting. The temperature must be 5,300 degrees before tungsten melts. That explains how it is possible to make these little filaments white-hot without destroying them.

On the table is a series of lamps with carbon filaments for comparison with the tungsten-filament lamps.

3 candle power	...........110 volts x	.1 ampere =	11 watts
6 candle power	...........110 volts x	.2 ampere =	22 watts
12 candle power	...........110 volts x	.4 ampere =	44 watts
16 candle power	...........110 volts x	.5 ampere =	55 watts
32 candle power	...........110 volts x	1. ampere =	110 watts

The effectiveness of an exercise like the foregoing will depend to a large extent upon what lessons follow this one and upon the educational ingenuity of the teacher. This work should be supplemented by a laboratory exercise in which students are given two dry cells, insulated wire, voltammeters, lamps, bells and motors. The laboratory project is to wire up a small electric lighting and power system and measure the current required to operate lamps, bells and motors.

It will, of course, be advisable and necessary to devote much of the next hour to general discussion and review work. The overhead line-wires will suggest an endless variety of experiments for purposes of review and for widening the students' electrical horizon. A second profitable lesson for purposes of general education may be centered around resistance wires and heating utensils.

THE THIRD LAW OF MOTION.

By J. O. Perrine,
Iowa State Teachers College, Cedar Falls, Iowa.

"To every action, there is an equal and opposite reaction." This short statement looks very unpretentious yet it is so subtle and full of meaning. As a student in high school and university, the writer failed to get a realizing sense of the meaning of this law. As a teacher first thrusting himself on the unsuspecting pupils, he always felt a heavy load lifted when this point in the book was passed.

To the illustrations that are usually given to exemplify this law, the average student readily assents. He can easily see that if one pushes against the wall with a certain force, the wall must push back with a reaction that is equal and opposite. It is perfectly evident to him that if one stands on the floor and thereby exerts a force on the floor, the floor must necessarily exert an equal and opposite reaction. Most students will agree that both boys pulling on opposite ends of a rope must pull with equal force. "Action is equal to reaction" runs glibly off the tongue. The texts give numerous examples of the above character with the result that both teacher and student congratulate one another on their appreciation and understanding of this fundamental proposition.

On the other hand, illustrative examples of the third law applied to problems in kinetics are not so numerous. In no case, where such examples are given, are the discussion and explanation lucid, accurate or satisfactory. The problem of a horse pulling a wagon is a typical problem of this type. When the horse pulls the wagon, then the wagon must pull back with an equal force. If the wagon pulls back equally, then one asks how is it possible for the wagon to move?

As a member of a group of a dozen or more graduate students, the third law applied to such problems frequently came up for discussion. It was interesting to note that no one of the group ever talked with thorough understanding or convincing argument. Rather it appeared that all were trying to get light on the subject. Several teachers of physics in good positions have been questioned on the above horse and wagon problem with the result that the person sometimes admitted that he didn't see how the third law could be true. One teacher said that he had told the pupils that no doubt the horse pulled ahead just a little bit more than the wagon pulled back.

No attempt to elaborate on the third law could be made without incorporating a discussion of the first two laws. Apply the first law to the wagon. The wagon is at rest, it tends to stay at rest, it is "lazy" and does not want to move. Indeed it is more than "lazy," it actually opposes any force that tries to move it, it "strikes back." If you do compel it to move, you therefore bring out this reaction. It resists, it pulls back since according to the first law it tends to stay at rest. In other words, it reacts because some force has made it move. However, once started, it likewise reacts against being given further acceleration, either negative or positive. If then, to repeat, it persists in a state of rest or uniform motion in a straight line, it will necessarily react with a push or pull when made to change that state.

The second law tells us how to measure this "striking back," this reaction mentioned or rather implied in the first law. If a body is made to move with an acceleration of $\frac{dy}{dt}$, the second law states that the body reacts against being given this acceleration by an amount proportional to its mass and its acceleration. As is usually stated, the force brought out by the fact that the body is made to change its velocity is proportional to time rate of change in momentum.

$$f \propto m \frac{dv}{dt}$$

If we choose our units properly we have

$$f = m \frac{dv}{dt} = ma$$

If, according to the first two laws, the body reacts with a force of *ma,* while it is moving, the third law completes the argument by saying that the "outside" force mentioned in the first two laws is equal to the one that the body exerts because it is moving. To reverse the order; the outside force acts and if motion results, immediately the reactive force comes into existence and is equal to the applied force. When we write $f = ma$, we think of f as the applied force and with no misgivings write it equal to *ma,* the reactive force.

By the very manner in which the unit of force, the dyne, is defined, we tacitly write down the third law. The force which is applied gives to a certain mass a certain acceleration. Necessarily then, the mass reacts with a certain force and we are

enabled to define the applied force by its being equal to the reactive force. Every time we write $f = ma$, we thereby say that the wagon pulls back just as hard as the horse pulls ahead.

When a horse pulls a wagon, the wagon pulls back as hard as the horse pulls ahead. If this is true how can the wagon move? The answer is, that it is not a question of how the wagon can move. The burden of proof is placed on the wrong phase of the problem. The wagon pulls back because it is moving. If it were not moving, that is of course with an acceleration, it would not pull back.

Suppose a man takes hold of an object which weighs 10 lbs. This body has the appearance of an object much heavier, so the man thinks he must lift considerably more than this, say 50 lbs. He exerts a force of 50 lbs. and what is the result? There is to be an equal reaction. The only way the 10-lb. object can resist with a force of 50 lbs. is to move. By its being given an accelerated motion, it resists to the amount of 40 lbs. and its own weight makes up the equal reaction of 50 lbs. A 50-lb. object can resist with a force of 50 lbs. Likewise a 10-lb. object can resist with a force of 50 lbs. too. The first object remains at rest, while the second must move. In the first case, the 50-lb. reaction comes from the pull of the earth, in the second case, the 50-lb. reaction exists because the 10-lb. mass moves with accelerated motion.

To the student of electricity, Kirchoff's second law really involves the same question as brought out by the third law of motion. Kirchoff's Law states that in any closed circuit the sum of the voltage is equal to zero.

Ohm's Law states that $E = RI$. Here we have Kirchoff's Law in its simplest form. $E - RI = O$. The impressed voltage, E, is opposed by an equal voltage, RI. Here then are two equal and opposite voltages. One asks, how is it possible for any current to pass when the above is true? Here again the answer is that by the very fact that a current flows, the counter voltage is immediately produced.

Consider a circuit having resistance and self inductance. In this case we write

$$E = RI + L\frac{di}{dt}$$

At every instant of time while the current is increasing in value, that is all the time that there is a $\frac{di}{dt}$, the applied voltage

is opposed equally by the sum of two others. One asks, how can any current flow at all; further, how can the current actually increase when the above is true? The answer is similar to the case in mechanics; by the very fact that the current is increasing or decreasing either, the counter voltage of self induction is brought into existence. This counter voltage due to inductance in a circuit, while the current is increasing, is analogous to the counter force an object exerts when the velocity changes.

We have a similar case in the operation of the electric motor. The applied voltage is opposed equally in this case by the sum of the so-called armature drop and the back voltage of the motor itself. This back voltage is made to exist by the very fact that a current passes through the armature coils. Hence the concept given in Kirchoff's Law is readily explained in this case.

When a permanent magnet is thrust into a close circuited coil, it is a well known phenomena that a current flows in this coil. Lenz's Law tells us that the current flows in such a direction as to oppose the action producing it. Assume that a magnet is being thrust into a coil with uniform velocity and a constant current is therefore flowing and a uniform field of intensity, H, exists within the coil. The motion of the magnet whose pole strength is m will necessarily be opposed by a force, Hm. Disregarding the purely mechanical forces involved which have been considered before, the opposing force Hm is exactly equal to the force which is exerted by the experimenter. If the moving magnet is repelled with a force exactly equal and opposite to that which the experimenter exerts, how is it possible that the magnet can be inserted at all? Here it is more easily understood than in the other cases, the force Hm exists because of the fact that the magnet is moving. Let the experimenter push harder and as a result the magnet moves faster; immediately a greater current is produced, a stronger field results and a force Hm opposes him equally and opposite as before.

The idea discussed in the previous pages also finds application in other phases of physics, including the transformer, differential equations and other more advanced branches of physical science.

If anyone will get as much straightened out in their understanding of the third law by reading this paper as the writer did in preparing it, then its preparation will have been worth while.

THE USE OF SPRINGLESS AUTOMATIC SCALES IN THE PHYSICS LABORATORY.

By L. P. Sieg, Ph. D.,
State University of Iowa.

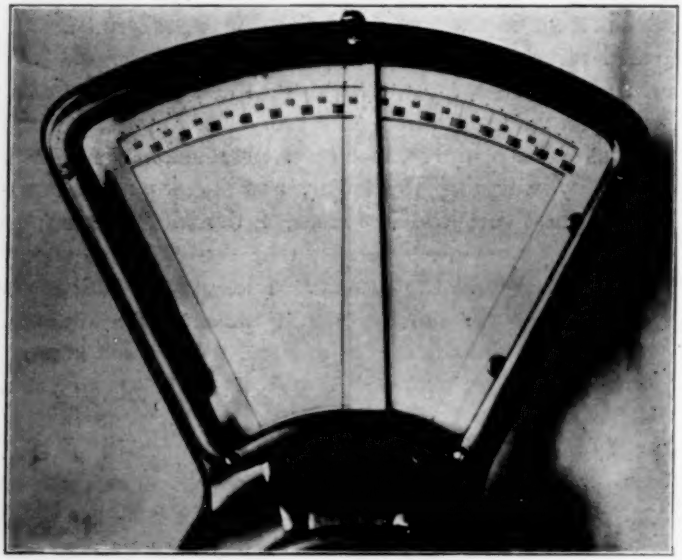
One afternoon, about a year ago, I chanced to step into one of the general laboratories where an assistant was in charge of a section of about thirty students. The particular field of work was in the subject of heat. I strolled about the large room seeing if everything was in smooth working order and on the lookout, as was my custom, for any serious flaws in the experiments or their operation, with an eye to the raising of the general efficiency of the work. The work seemed to be going excellently; the students were going about their tasks in a serious and purposeful manner. All seemed well until I came to the balances and here I had abundant food for thought. There were two of the typical equal-beam balances in use—a third there was, but more of it later—and in front of each were a number of impatient students waiting their turn. Here was a serious flaw in an otherwise excellent machine. I stationed myself near the balances and observed. A student—let us call him A—came up to weigh an empty calorimeter cup with its stirrer. He had to wait until three others ahead of him had finished their weighing, each taking by actual test an average of one minute for the weighing. A then weighed his cup. Four minutes had now elapsed. Note here that the greater part of each one of these minutes was used in getting the correct tenths of a gram, and with some erratic draughts in the laboratory, this was a tedious process. A then returned to his table, partly filled his cup with hot water, and came up to the balances to weigh again. This time only two students were ahead of him at the second pair of balances, and only two more minutes elapsed before he was ready to weigh. Things went well until he sought the fifty-gram weight, but some careless student had misplaced this. He was able, however, to get over the difficulty by selecting other weights, but three more minutes had gone by. Then he went back to get his ice to drop into the calorimeter and so he continued the experiment, and repeated the whole thing twice. The further details need not be entered into. Suffice it to say that he spent at least one-half of his time either weighing or waiting to perform his weighing. This time so spent, thought I, could not be justified for a moment from any standpoint whatever. The

weighing practice surely was not needed—the stupidist common laborer could do nearly as well.

The above chance observation decided me that steps must be taken at once to improve this condition. Two courses seemed to present themselves. First to buy four more balances with necessary weights, at a cost of about $14.00 for each outfit. Second, increase the rapidity of weighing by adopting some other form of balance. The first course carried with it two objections. First, the time of a weighing would still be unnecessarily long; and second, there was the continual danger of lost and inaccurate weights. Let me state here parenthetically that the third pair of balances mentioned above were out of commission because some zealous research student had borrowed the weights a few hours before. Even if the set of weights is not carried away outright, every instructor knows how difficult a matter it is to keep the weights intact. It can be done, but usually at the cost of ceaseless inspection. In a large laboratory, where there are several different instructors during the week, the confusion, due to divided responsibility, becomes a serious matter.

For the second course—to change the style of balances—I received splendid inspiration a day or two following when my grocer was selling me some sugar. He shamed me and my laboratory by weighing the sugar, and incidentally noting the price in about five seconds. Here, I thought, was a possible solution. Why can't we physicists take this excellent suggestion from the butcher and the grocer? I at once wrote to several dealers in such scales—for they call them scales—and had for my pains a vast amount of cold water thrown on my spirits. One well-known firm wrote—"Would advise that we cater exclusively to the butcher and grocer trade, and we could not undertake to make a chart graduated as you wish it." Only one firm responded favorably, but the price asked, $60.00, for one of their smallest scales seemed out of the question. Finally, after a deal of urging on my part, one firm was reckless enough to lend us, for trial, one of their small, springless candy scales, made up with a temporary chart reading 0-1,000 grams. The chord of the arc of the graduation was 155 mm., the radius of the arc was 190 mm., and its length was 160 mm. There were in all one hundred divisions each 1.6 mm. long. Thus the smallest division represented 10 grams, and it remained to be seen whether a student could estimate with accuracy to a tenth of a division, and whether the working of the scales was reliable enough to war-

rant this accurate estimate. The scales proved by numerous tests that the fine hair line indicator would come to the same reading within one-fifth division no matter how many times the object was placed on the pan, and no matter in what manner the weighing was accomplished. The student easily learns to estimate a small space to tenths of a division, and his error ought never be greater than one-fifth of a division. Here then was a scale reading up to 1 kg. with an accuracy of one-fifth of one per cent for full scale, and of course with a lessened accuracy for less weights. The time of weighing is easily no more than five seconds, and so this one scale ought to serve as many students as six of the older type of balances. If the objects weighed were too light for these scales, I did not seek to get more delicate scales, but rather to increase the size of the objects experimented on. We found that a few experiments still needed the refinement of the old balances, so one pair was left in commission.



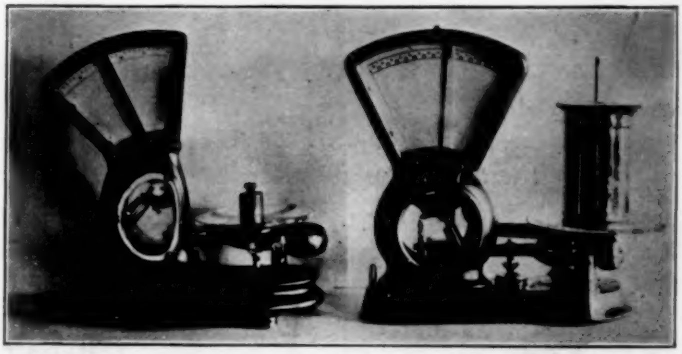
Close view of chart of 1500 gram scale. The reading is 719 grams with a possible error of $\pm$ 1 gram. This accuracy is superior to that needed for most elementary experiments.

Needless to say the experiment proved a pronounced success. There was no delay whatever in the work in heat, in which subject balances are most in use, to be charged against the pair of scales, there were no misplaced weights, and lastly the accuracy

was all that was needed. Think of working an experiment in specific heat by the method of mixtures and determining the weight of the filled calorimeter cup to be say 325.6 grams, when the total rise in temperature is only about 5°, with a possible error of 10 per cent! All physics teachers know that our weighings are made with more supposed accuracy—I say supposed, when I think of the abraded and corroded weights—than almost any other of our measurements.

Summing up then the above arguments in the case of automatic scales vs. beam balances:

a. The automatic springless scales will enable a student to make a weighing in about five seconds—the beam balances take from thirty to sixty seconds.



The scale on the left has a 500 gram range; that on the right a 1500 gram range. The hair-line pointer makes exact readings possible. Time of weighing under five seconds.

b. The automatic scales are sufficiently accurate for any ordinary physics experiments.

c. The automatic scales will easily do the work of six common balances, i. e., $35 to $40 expended on an automatic scale will do the work of $80 in the common balances.

d. There is no occasion in the use of the automatic scales, to be concerned with lost, abraded or corroded weights. The calibration of the scales can be checked from time to time by means of a set of standard masses.

e. Even in a small laboratory, where only one common balance is needed, the automatic will save enough of the students' time to make the extra cost well worth while.

In closing, fearing that some will think that here is a tre-

mendous disturbance over a small matter, I must emphasize that our whole teaching of physics is made up of small matters, and it is only by detecting and correcting these small faults, that we shall be doing the much-needed work of improving our teaching on this subject. The day will never come when we can say that our laboratories are perfect.

Note: The writer feels that his best thanks are due to the Detroit Automatic Scale Co., Detroit, Michigan, for their generous coöperation in the development of automatic springless scales for the use of physics laboratories. They are now prepared to furnish two types of scales with flat weighing platforms; one reading to 1,500 grams, and one to 500 grams. Anyone interested can learn more details by writing directly to the company.

INTERPRETATION OF RESULTS IN CHEMISTRY TEACHING.

By Jessie Caplin,
West High School, Minneapolis.

A chemist employed in the technical testing of the raw material and finished product of a manufacturing plant visited in Chicago a school where the chemical methods used in that industry are taught in three months!! In discussing the value of such a course, the chemist said that it might be advantageous for the son of a man owning a small plant; but that for a chemist to be of real value, he must be able to interpret results.

What is the high school teacher teaching chemistry for?

When a large proportion of his pupils go to some college and continue work in the subject, he is foolish if he does not consider the freshman work demanded as well as entrance requirements. When newspapers and magazines contain in almost every issue items in which chemistry and its applications are so plainly concerned as to be headlined, the teacher is losing an opportunity of enhancing the interest in his subject and increasing its value if these items are not in some way related to class work. When the practical value of chemistry to industry is increasing each day, a statement of the varying phases of work to be done will often make the high school class a small but valued feeder through the university to the industrial laboratory.

The value of high school chemistry to those who go no further with the subject or with formal education is important. It can

give training in careful observation, exact statements, and logical thinking. This should be the underlying purpose whether preparation for the university, interest and judgment with regard to its commoner applications, or the turning of a very few to its specialized use in commercial laboratories is the aim.

The high school teachers of chemistry should take the time and make more effort to test their teaching and *interpret the numerical results*. The chemistry teacher is most favorably placed. 1. He has seniors—many of the undesirables have been weeded out and experience has taught many of the others how to apply themselves to study. 2. The subject is often elective—few have to take it who do not wish to. 3. Success in it is not dependent on proficiency in other courses in science and mathematics. 4. This last, together with the novelty of the subject, discourages prejudice, while the subject matter and experiments arouse immediate interest and enthusiasm.

When the test comes—the daily quiz, the monthly review, the "final"—there should be for the teacher two points of view in making the questions, and in using the results—first, to determine the character of the raw material—the preparation of the pupil for further work; second, the nature of the product—how well the teaching has been done. The two lines of vision converge at a focus determined by what the teacher considers important in the unit of work tested and how sharp an image the teacher desires.

Now as to the testing and interpretation of results. The teacher has to depend on university reports and occasional conferences with pupils back from college to determine how satisfactory his preparation has been to the university and to the pupil. The teacher can grade *his* efficiency in this regard. In case of failure in college, a reading of the text used, an investigation of the quiz questions set, together with a conference (often necessarily written) with the university teacher and the backward pupil will cause the high school teacher to change the emphasis and the rate of progress. Successful pupils can often give valuable information as to where the preparation and method were helpful.

The degree in which the pupil relates his class chemistry to his reading and everyday life can be estimated by reports on what is read, the bringing in of clippings and titles of interesting articles, and by the amount of material brought in for testing. The interest shown by the pupils during a discussion will give a

general estimate as to the value derived; and the use of illustrative material and reading as the basis of English themes and commencement essays will give a sufficient bonus for extra work.

In the classroom what is the recitation worth to the individual pupil and to the class? Is the pupil ready for the next work? Can he see and make a connection between the new work and that of the past? The teacher who cares about these things will surely stress certain parts of the new lesson because of its relation to the old one; drill on other parts because it is a necessary foundation for further work; discuss other points because the foundation must be as broad as possible. The teacher will sum up with a very definite series of questions designed to test these results: to see whether the relation to the old topic is understood, how well the new work has been started, how much of the additional information has become a part of the foundation. These results should be checked as soon as possible by written work. If a large percentage fails on one or more questions, it is usually the teacher's fault and the work must be repeated. In the case of individual pupils the mistake should be corrected in writing and the teacher should try to find the cause of the error.

In the monthly quiz, it is presumed (1) that the teacher wants additional aid in determining the grade to be sent to the office and to the parent; (2) he wants to find out whether the pupil knows certain topics studied. *The more definite the teacher makes the assignment for the review, the longer the pupil will work in preparation. He will drill himself on what the teacher, from experience, considers most important;* (3) from these results the teacher will plan the succeeding work to fill in crevices he has left, and strengthen the foundation, to build anew and better because of past experience.

RAILROAD MAPS.

At the November meeting of the Central Association of Science and Mathematics, a resolution was introduced at the general meeting and unanimously passed, that it was the sense of that Association that Congress take some action to correct the railroad maps of the United States, which everyone who knows anything about the country realizes are absolutely untrue in most instances. It was suggested that the railroads be compelled to issue their maps in accordance with the United States government geological survey map. It was also suggested that all educational associations and teachers be invited to co-operate in bringing about a remedy for this evil. It was voted that a committee of fifteen be appointed to take the matter under consideration and devise schemes for the best course of action.

MATHEMATICS AND EFFICIENCY.[1]

BY FLETCHER DURELL,

Lawrenceville, N. J.

Much as the word has been overused and abused, the fact still remains that efficiency is the dominant thought of the age just as evolution was the controlling idea a generation ago. Now the interesting fact for us as teachers of mathematics is that efficiency is essentially a ratio. It is the ratio between product and cost; or between output and input. Thus, if a farmer receives $1,200 for a crop of wheat which has cost him $800 to raise, his efficiency in this process is $1200 ÷ $800, or 1.5, that is, 150 per cent. But in this connection we must immediately make a distinction between absolute and relative efficiency. Let us take the case of a dry farmer out West. Let us suppose that in a given case he spends $800 in raising a crop for which he receives $600; but under the given conditions the normal or bogey crop would be worth only $400. Hence his absolute efficiency is $600 ÷ $800, or 75 per cent; but his relative efficiency (that is, his efficiency relative to what is average or normal) is $600 ÷ $400, or 150 per cent. Similarly, the largest and best steam engines utilize, say, 18 per cent of the energy in the fuel consumed by them; that is, their absolute efficiency is 18 per cent. But when compared with a small crude steam engine which utilizes only $1\frac{1}{2}$ per cent of the energy in its fuel, they are $18 \div 1\frac{1}{2}$ or 1200 per cent efficient.

The above ideas seem simple and elementary enough. Yet considerable, and often costly, confusion in regard to them exists even in the minds of those who are regarded as efficiency experts. For instance, I have a letter from a man prominent in efficiency councils, who apparently denies or cannot grasp the distinction between absolute and relative efficiency. At least, in his letter he specifically asserts that it is impossible to have an efficiency of over one hundred per cent. Mathematics and mathematical training can do a real service by giving a firm, clear grasp of these fundamental ideas.

We are now ready to go a step further and consider the case of compound or resultant efficiencies. Here it is of prime importance to make a distinction between additive and multiplicative combinations of individual unit efficiencies. As an example

[1]Read May 1, 1915, before the Association of Mathematics Teachers of New Jersey.

of an additive combination of efficiencies, we may take the following:

In a given year, the efficiency of a farmer's wheat crop was .80, of his potato crop, .90, and of his hay crop, .70. Find his average efficiency for the three crops.

Evidently we have as the solution $\frac{.80+.90+.70}{3}=.80$, *Ans.*

An illustration of the multiplicative combination of efficiencies is as follows:

In a certain factory a given lathe is being run for one-half its possible operating time, at one-fifth its possible speed, and with a cutting point only one-sixth as wide as possible. Find the efficiency of this machine.

Since each of the factors involved acts, so to speak, upon the resultant of the others, the required solution is $\frac{1}{2}\times\frac{1}{5}\times\frac{1}{6}=\frac{1}{60}$, or the given machine is less than two per cent efficient. In the recent earnest search for increased efficiency in factories, actual instances like this have been found not infrequently (see *Tuck School Conference on Scientific Management*, p. 166). In fact, it is not going too far to say that many a business man has been ruined by not realizing the multiplicative relation that frequently exists between different efficiency factors in his business. He thinks that certain losses and wastes are small and individually negligible or at least not worth the trouble and expense involved in their eradication. He does not appreciate the fact that these elements of waste act on each other in successive multiplicative or tandem style, and thus often form a highly destructive product.

The above distinction between additive and multiplicative combinations of unit efficiencies is also useful in analyzing and unravelling more complicated cases. In this connection at the outset we must consider that kind of average or additively combined efficiencies where the elements in hand are of unequal value; or, as it is sometimes termed, are weighted. Thus, in the preceding example of an additive combination, if the wheat and hay crops are each twice as important as the potato crop, the average efficiency is

$$\frac{2\times.80+.90+2\times.70}{1+2+2}, \text{ or } .78.$$

Similarly, if a pupil studies reading, history, geography, and arithmetic, and the weights assigned to these subjects are respectively 1, 2, 3, 4; also if the pupil's grades in these subjects

are .80, .85, .90, .85, the pupil's average would be

$$\frac{.80+2\times.85+3\times.90+4\times.85}{1+2+3+4}, \text{ or } .86.$$

We may get at this result by another method which it is important to consider. Let 100 be the mark which a pupil is to receive if doing perfect or ideal work in all studies; then in accordance with the above set of weights, the maximum or perfect work in each of the studies specified above would be 10, 20, 30, 40, and teachers might have saved labor in making out the above average grade by handing in the following marks in their several departments: 8, 17, 27, 34; which added together give 86 as the pupil's efficiency or standing with respect to the assigned ideal of 100.

The method which has just been described is essentially the score card method of determining efficiency or of grading objects of a given class. This method is being more and more widely used, as in grading babies at a baby show, or in grading or rating horses, milch cows and farm animals of all kinds, or even farms themselves. In this connection, it should be observed that the score card method of combining elements of efficiency regards these elements as fundamentally additive in relation, and that this is probably an error. Thus even in the case of the farmer, if his operations be considered in their connections for a number of years, they will be found to be more and more dependent on each other in multiplicative ways. For instance, the character of the hay crop for one year determines the kind of sod which is to be ploughed under later, and thus is a factor in the efficiency of the next corn crop. So the factors of efficiency in a horse or in a workman or in an overseer or in the highest type of intellect are related more or less multiplicatively according to various circumstances. The analysis and unravelling of these complicated cases of interrelated elements of efficiency open a large field for mathematical study. In fact, we may raise the question whether we do not find in the investigation of the various ratios involved in efficiency and also in the study of the different more or less complicated ways in which these ratios may be combined, the beginnings of a branch of mathematics with daily applications to every department of life and activity.

But we may take up the relations of mathematics and efficiency from another and equally important point of view. If the ques-

tion be raised at any time as to why a certain great man has been successful, sources of efficiency like the following are usually assigned—energy, shrewdness, courtesy, persistence and reliability. These categories are well worth noting and studying, but they have the disadvantage that they are not easily organized together as a whole; they are not adapted to progressive and systematic study and mastery. I wish to suggest certain other categories or sources of efficiency which underlie or precede the traditional sources, which are also capable of progressive treatment and which lead up to and give new force and added value to the conventional elements of success.

The first of these progressive and systematic, and therefore semimathematical, sources of efficiency which I would mention in this connection is reuse or repeated use. An instance of a person who practices reuse is a cook who has devised a good dish, and then uses it again and again on different occasions, perhaps gives it to others to reuse, thus saves different persons the labor of devising new recipes and adds to the happiness and health of mankind. Other examples of reuse are the repeated uses which have been made of the improvements which Watt made in the steam engine, or which have been made of the work of the inventors of the alphabet.

It is at once evident that various mathematical categories may be applied to reuse. Thus, the idea of number is here directly present, and the greater the number of times an idea or object is repeatedly used, the greater, usually, are the resulting advantages. Since an object can be reused in whole or in part, the idea of quantity or magnitude may also occur. An example of partial reuse is that of the continued utilization of a pair of shoes by having them half-soled. The ideas of dimensions and geometric form are also often useful aids in developing a given case of reuse to its maximum fruition. For example, a traveling crane that is used in various places by being moved back and forth along a beam constitutes what may be termed a species of linear reuse. If a crane is reused in different places by being rotated on a fixed point, we have a case of circular reuse. If an idea spreads and is reused in a surface, as throughout a given country, we have areal or two-dimensional reuse. Similarly, cases of reuse may be three-dimensional in character, or may have any geometric form.

The second of the semimathematical sources of efficiency which I would mention is the group principle. A familiar in-

stance is that of teaching pupils in groups or classes and not individually, wherever possible. I recently heard a graduate of a woman's college complain bitterly of the impractical character of the instruction which she had received because, after she was through college, she did not know enough when making a number of similar garments to cut them out, several at a time; that is, in groups. But the group is essentially a mathematical concept. It is vitally concerned with number; the unit objects composing a group may be arranged in a line, or as a two-dimensional, or three-dimensional aggregate; a group may have all sorts of shapes, according to various conveniences or advantages. Hence, the pupil who by a thorough study of mathematics is master of the ideas of number and form should have distinct advantages in using groups in the most effective ways.

But individual groups may be combined to form systems of groups with correspondingly increased advantages. If we conceive of a certain set of groups as combined in such a way that each element in one group has useful or functional relations with several of the objects in at least one of the other groups, we have what is an essentially mathematical conception of the essence of every organization or system. Now a set of related groups may be arranged in a line, an example being the cards in a card index. Or the connected groups may take a fan-shaped form, as with the commander-in-chief, generals, colonels, minor officers and privates of an army; or they may constitute a rectangular array as with a set of pigeonholes. Or they may form a three-dimensional aggregate as with the various divisions and subdivisions of space and materials in a factory. The point that interests us most in this connection is that a thorough study of mathematical categories like number, quantity, dimensions and space enables a person to develop something like an adequate conception of the vast profusion of forms which this mathematical essence of organization may assume. The business man who has this large preliminary conception finds it correspondingly easy to select the particular type of organization best fitted to his needs in any given situation.

Moreover, a necessary aid in dealing with these larger and more complicated aggregates of groups or group systems is symbolisms. These symbolisms are of two principal kinds—(1) geometric, as with charts, graphs, diagrams; (2) algebraic, as by the use of numerical-alphabetic symbolisms, or even by ordinary language. Hence, a thorough grasp of these two chief

kinds of symbolism as inculcated by the study of mathematics is of prime importance in this connection.

Before closing, it should also be pointed out that a thorough grasp of the semimathematical categories of efficiency described and illustrated above and other similar semimathematical instruments which might be mentioned, leads to, or even compels a close study and aggressive use of the customary or conventional sources of efficiency. Thus, for instance, to obtain the maximum reuse of an object or idea, we must usually exercise a high degree of will power, accuracy, honesty, tact, order, system, energy, and similar means of obtaining results.

Hence we arrive at the following general conclusion. In the development to its full fruition of the leading thought of the age, mathematics may be an important aid; in this process, mathematics is made into something of daily application and use in all the details of life in new ways.

CALCULATION OF THE STRENGTH OF ELECTRIC CURRENTS.

Probably the most accurate method for the determination of the value of the strength of an electrical current in absolute measure is by means of the Rayleigh current balance, in which the current to be measured is passed in series through two parallel circular coils of unequal radii, one of which is suspended from the beam of a balance. The distance between the planes of the coils is varied until the force of attraction between the two coils is a maximum, and the value of the force is obtained by adding weights to the other arm of the balance until its equilibrium is restored. Since the maximum force obtainable depends on the ratio of the radii of the coils alone, and not on their individual dimensions, it is only necessary to determine further the ratio of the radii of the coils, and this may be done with great accuracy by electrical means.

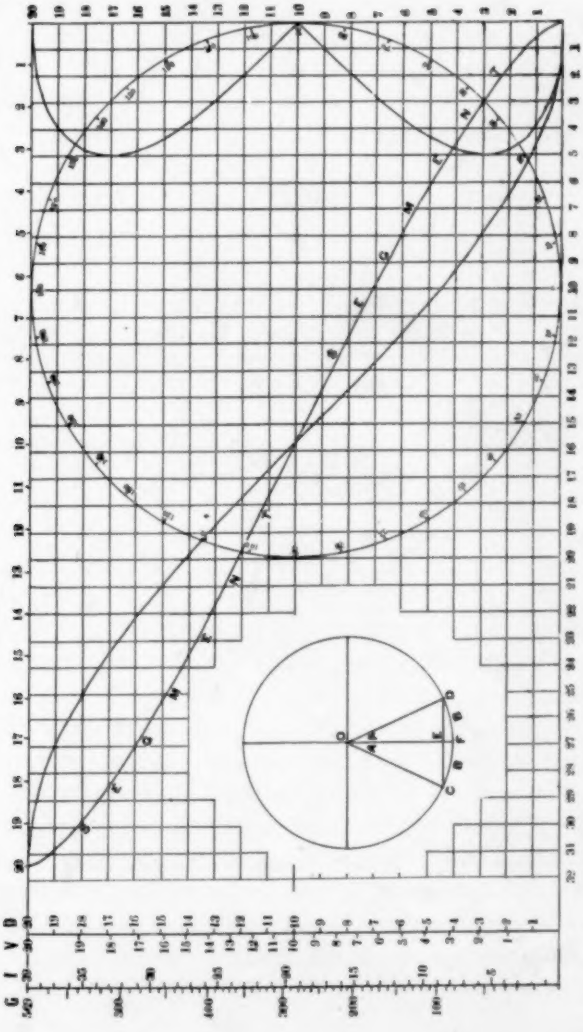
The constant of the instrument, that is, the maximum force per unit current for the coils in question, has been obtained in the past by interpolation between values of the force, calculated for various assumed distances of the coils, in the neighborhood of the critical value for which the force is a maximum. For, although the general formulas of Maxwell and Nagaoka give the value of the force for any two given coils, at any assumed distance, with great accuracy, no formula has been heretofore published for calculating at what distance the force becomes a maximum. To supply this lack there is derived in a paper just published by the Bureau of Standards, Department of Commerce, entitled *The Calculation of the Maximum Force Between Two Parallel, Coaxial, Circular Currents,* a formula which gives the critical distance as a function of the ratio of the radii. The latter part of the paper is devoted to the development of methods for facilitating the calculations. The formulas are illustrated by numerical examples and tables, and the new formulas are shown to give results in agreement with those derived by more indirect and laborious methods of interpolation.

Copies of the publication, *Scientific Paper, No. 255,* may be obtained on request of the Bureau of Standards, Washington, D. C.

GAGING A HORIZONTAL CYLINDER.

By William F. Rigge.
Creighton University, Omaha.

The actual amount of liquid present at any time in a horizontal cylindrical tank may be gaged very rapidly, if we make use of a graphic process such as is furnished by the diagram in this article. As the problem evidently resolves itself into finding the relation between the height and the area of the segment of a circle, we generalize it by taking the radius of the circle as unity, and then computing and plotting the areas corresponding to various heights.



From the small circle inserted in the diagram we see that the area of the segment CDF is evidently the area of the sector O-CFD diminished by that of the triangle OCD. Both of these latter areas are functions of their semiangles A.

The area of the sector O-CFD is equal to half the length of its arc times the radius, and since the radius is unity, it is simply B, half the length of the arc CFD. We find this value of B from A in a suitable table. If we have no such table, we take the 360th part of the circumference 2×3.1416, and multiply it by the number of the degrees in A.

The area of the triangle OCD is $ED \times OE$, that is, sin A cos A, or $\frac{1}{2}$ sin 2A. Turning the triangle around so that either OC or OD is its base, we see at once that its altitude is sin 2A, and hence its area half that much.

In the following table the values of A are given for every ten degrees. This is sufficiently accurate for the purpose.

Semiangle .... A	0°	10°	20°	30°	40°
Height .... EF	0.0000	0.0152	0.0603	0.1340	0.2340
Sector .... B	0.0000	0.1745	0.3491	0.5236	0.6981
Triangle .... $\frac{1}{2}$ sin 2A	0.0000	0.1710	0.3214	0.4330	0.4924
Segment = Sector — Triangle	0.0000	0.0035	0.0277	0.0906	0.2057

Semiangle .... A	50°	60°	70°	80°	90°
Height .... EF	0.3572	0.5000	0.6580	0.8264	1.0000
Sector .... B	0.8727	1.0472	1.2217	1.3963	1.5708
Triangle .... $\frac{1}{2}$ sin 2A	0.4924	0.4330	0.3214	0.1710	0.0000
Segment = Sector — Triangle	0.3803	0.6142	0.9003	1.2253	1.5708

Semiangle .... A	100°	110°	120°	130°	140°
Height .... EF	1.1736	1.3420	1.5000	1.6428	1.7660
Sector .... B	1.7453	1.9199	2.0944	2.2689	2.4435
Triangle .... $\frac{1}{2}$ sin 2A	0.1710	0.3214	0.4330	0.4924	0.4924
Segment = Sector + Triangle....	1.9163	2.2413	2.5274	2.7613	2.9359

Semiangle .... A	150°	160°	170°	180°
Height .... EF	1.8660	1.9397	1.9848	2.0000
Sector .... B	2.6180	2.7925	2.9671	3.1416
Triangle .... $\frac{1}{2}$ sin 2A	0.4330	0.3214	0.1710	0.0000
Segment = Sector + Triangle....	3.0510	3.1139	3.1381	3.1416

In constructing the above table, we first find B corresponding to A. Next we take out $\frac{1}{2}$ sin 2A from a table of natural sines, and from the same table, we also find EF by subtracting cos A from 1.0000. We see that B increases uniformly, that $\frac{1}{2}$ sin 2A consists of only four or five values which repeat themselves, and that E F for angles greater than 90 degrees is found by subtracting the value for its supplement from 2.0000, or, more rapidly, by adding 1.0000 to the cosine of its supplement.

We now put our computed quantities into graphical shape. In the diagram the radius of the large circle is our unit. On

the vertical scale to the right every tenth of this unit is noted and indicates the height of the segment or the depth of the liquid present in the cylinder. The decimal points are everywhere omitted. Hundredths of the radius may be easily estimated and even the thousandths, if we use co-ordinate paper with ten lines to the inch and make our radius equal to ten inches. We may then attain to an accuracy of one-tenth of one per cent.

The horizontal scale below the diagram uses the same unit as the vertical one. In order then to plot our values, let us take A = 50 degrees, for instance, for which we found E F = 0.357 and the area of the segment 0.380, the fourth decimal not being needed. Starting from the right-hand lower corner of the diagram we run up vertically the distance 0.357, and then horizontally to the left to 0.380 and there mark one point on the curve called "Segment" on the diagram. This point is, of course, on a level with the 50 degrees on the circle. Other values are similarly plotted, and the points joined by a smooth curve.

The diagram here presented also gives the areas of the sectors and triangles corresponding to the heights of the segments. The long curve nearest the "Segment" curve and seemingly its mate, indicates the areas of the sectors, while the half 8 near the vertical scale shows the areas of the triangles. These curves have not been named on the diagram in order to prevent confusion between the sectors and segments. They are given merely for the sake of geometrical completeness, and are not needed for our purpose.

The areas on the bottom scale are in terms of the radius, as said before. In order to use this diagram for a circle whose radius is not unity, we have but to multiply the indications of the vertical scale by our actual radius, and those of the lower horizontal one by its square.

The upper horizontal scale is in decimals of the half-area of the circle. This scale is reproduced in terms of the radius on the vertical scale V D shown near the left of the diagram, V meaning the half-volume or area and D the depth. For example, 0.3 of the half-area of the circle, or 0.3 of the half-volume of our cylindrical tank, as shown on the top scale, when traced vertically downward on the diagram to the "Segment" curve, shows a height of the segment or depth of the liquid of 0.416 as read on the right vertical scale. Following this horizontally to the left, we find 0.3 of the half-volume reading 0.416 of the

radius in depth. If we prefer to take the diameter and the total volume as our units, we need but omit all the odd numbers on the diagram and divide the even ones by two.

As the diagram here given is perfectly general, it only remains for us to see how to apply it to any particular case. Thus suppose we have a cylinder 39 inches in diameter and 110 inches long. We begin by dividing our diameter or double radius into 39 equal parts. This is done on the vertical scale to the extreme left under the letter I (inches).

We next mark the gallons. Knowing that our given tank holds a volume of 568.8 gallons, we set 569 on our slide rule opposite to 2.00, and use the upper horizontal scale, or better, we set 569 opposite to 3.142 and use the bottom scale of our diagram and the graduations of the co-ordinate paper. Then the slide rule will give us all we need. For example, let us take 100 gallons. On the slide rule we find 100 opposite to 553. Finding this 553 on the bottom scale, we run vertically up to the "Segment" curve and then horizontally to the extreme left and mark the 100 gallons. If we have no slide rule, we must use the proportion $\frac{568.8}{3.142} = \frac{100}{x}$, or $x = \frac{3.142}{568.8} \cdot 100 = 0.553$, that is, we multiply the gallons by $\frac{3.142}{568.8}$. Noting the inches and fractions corresponding to our gallon numbers, we transfer the whole scale GD to true inches and thus correctly calibrate our gage glass or rod.

A purely graphic way of calibrating the gallon scale without a slide rule and without logarithms, is to divide the horizontal scale of the diagram into 569 equal parts. This is best done by computing the reading on the lower scale for 500, that is, by finding $x = \frac{3.142}{569}\ 500 = 2.762$, and then dividing this distance into five parts for the hundreds, and these again into ten parts for the tens, or even into hundred parts for the single gallons. The gallons being thus evenly spaced on the horizontal scale, we run up (or down) vertically to the Segment curve, and then horizontally to the left of the scale of inches. With a slide rule the actual graduation of the horizontal scale into gallons is, of course, not necessary, and has not been done in the diagram.

REFORM IN THE TEACHING OF MATHEMATICS.

By Professor G. A. Miller,

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Many of the proposed mathematical reforms relate to changes in the subject matter employed in teaching, such as an increased emphasis on the concept of functionality and an earlier use of the notion of derivative. There is, however, a more important reform through which probably most mathematics teachers have to pass more than once; viz., the reform which leads the teacher to present the subject matter from the student's point of view instead of from his own more advanced point of view. The easiest way of doing a thing from the standpoint of the mature teacher is frequently not the easiest way from the standpoint of the immature student.

It is said that a very successful merchant who began his career with a small store, started out by spending considerable time on the streets and in the parks for the purpose of observing what things were most commonly worn by various classes of people and then he ordered goods for his store accordingly. Similarly, the young teacher who expects to make a real success as a teacher should try to find out what methods the students actually like and he should then aim to present his work accordingly. This is especially true in regard to the teachers in the high schools and in the colleges since these teachers are in the greatest danger of overlooking the difference between their own point of view and that of their students.

In the most elementary work the difference between the student's point of view and that of the teacher is so wide that few fail to observe it, but as the student advances, this difference clearly decreases and the danger of overlooking it increases. As long as there is a wide difference between the mathematical maturity of the student and that of the teacher, there remains the danger that the methods which appear easiest to the teacher are not those which appear easiest to the student. Hence the most scholarly teachers have to exercise the greatest care in presenting a subject. If they exercise this care they naturally become the best teachers, otherwise they sometimes become the poorest.

The American high school teachers are in especial danger at present along this line in view of the fact that so many of them begin their teaching duties with an inferior preparation, and then make up this deficiency by work in Summer Sessions. They are

naturally anxious to give to their students the broader and inspiring viewpoints which came to them during these later stages of preparation, and often fail to observe that these viewpoints are of especial interest only to the more mature minds. General views are of more interest than special ones only after the mind is sufficiently mature to comprehend their generality.

High school teachers need to be warned not to transplant directly into their high school classes the methods employed by their Summer Session instructors. Even those methods which were most helpful and inspiring in the classes composed of teachers may prove to be almost total failures when transplanted into the high school. There is not one ideal method of presenting a subject, but there are generally as many such methods as there are different grades of students to whom the subject is to be presented. A good teacher for one class of students may be a very poor teacher for a different class. This is especially true of teachers who lack adaptability.

The main function of a normal school training in mathematics is to habituate the prospective teacher to look at elementary mathematics from an unnatural point of view. Difficulties which have long ceased to exist in his mind must be revived and exhibited in the light of the youthful mind, and the deeper difficulties which appear to the mature mind should be entirely overlooked. Even the natural numbers do not cease to present new difficulties after each conquest of the older ones, and thus they furnish problems to the most advanced mathematician as well as to the child who is just beginning to count. They constitute the fountain of all mathematics[1] and involve an inexhaustible supply of secrets for mathematical students in all different grades of advancement.

Although the high school teacher of mathematics is not required to approach his subject with the same degree of unnaturalness as the elementary teacher, yet he, too, has to revive difficulties which no longer exist in his own mind. The comprehensive views which unite and clarify for him much of the elementary mathematics must be kept in the background, and individual and somewhat superficial difficulties must be magnified and explained. Results that could be embodied in a sentence have to be dilated into pages in such a way that the difficulties which they present to the beginner are met by easy steps.

[1] *Der Urquel aller Mathematik sind die ganzen Zahlen*, Minkowski, *Diophantische Approximation*, 1907, Preface.

Hence the high school teachers, as well as the teachers of freshman mathematics in the colleges and universities, are continually called upon to reform from the natural to the unnatural way of looking at the subjects which they are expected to teach. This reform is often simplified by recollections of the impressions of early life but it is a disturbing element to those who are seeking to advance in their own mathematical attainments, and it helps to explain the fact that those who have taught elementary mathematics successfully for a long time usually acquire a habit of looking at things which is more apt to retard than to expedite the study of advanced mathematics.

Many teachers depend upon the textbook to supply what would appear at their own stage of advancement to be unnecessary amplification, and make little conscious effort to adapt their mode of reasoning to those whose outlook is much more limited than their own. While a good textbook, embodying the point of view of the student instead of that of the mathematician, may render useful service, it can clearly not take the place of a sympathetic teacher who presents the subject matter from various points of view and with various degrees of generality, starting with what is in easy reach of the student.

One of the chief delights of an instructor who has the privilege to give advanced courses in mathematics is that he is expected to reason naturally in presenting his subject to the students taking such a course. He can reason before such students like a mathematical man instead of like a mathematical child. The reasoning employed by the teacher should, of course, always be somewhat beyond that which would be naturally employed by the student in order to train the latter for more and more comprehensive views, but this difference must be moderate. The successful teacher of mathematics is not satisfied when he can solve readily all the problems of the lesson but he is anxious to present fundamentals so clearly that his students can solve their own problems.

One of the greatest dangers in the teaching of elementary mathematics is an undue emphasis on the formal logic side. The mature teacher naturally assures himself that every step taken by him is in accord with a law which he can readily formulate. The immature student, on the other hand, is usually much more easily satisfied as regards the correctness of his result, and is more interested in the mathematical content than in the formu-

lation of the logical processes involved. Questions relating to the smallest number of postulates involved belong to a comparatively late stage of mathematical developments. There are students who know a great deal about a subject without being able to formulate a correct definition of this subject. On the other hand, there are those who know the definitions well but very little beyond that. Great mathematical advances were made by men who were ignorant of the fact that there are continuous curves which have no tangent lines, and who even assumed that such curves did not exist.

While the changing conditions doubtless call for reform in the subject matter selected for the early courses in mathematics, the teacher of this subject should never forget his tendency to generalize more and more as his knowledge increases and hence the need of reforming his natural way of looking at things when he presents mathematical matters to those whose attainments are very limited. If this need is recognized, the more advanced knowledge on the part of the teacher should tend towards simplicity and clearness in dealing with elementary subjects. If it is not recognized, this more advanced knowledge may actually be detrimental in his teaching of elementary subjects.

In trying to improve his methods of teaching, the young teacher cannot be too strongly advised to consider the views of his students and the actual results attained by them rather than some preconceived notions which seem to be in accord with a few accepted psychological principles. In this respect he may perhaps profit by the development of the subject of mechanics. During the eighteenth century the investigators in this field were generally dominated by the ideal to deduce everything from a few assumed general principles, but in later times it became customary not to encumber the progressive knowledge of facts by deductive principles based on a limited number of phenomena. Instead of seeking to subject all phenomena to a single physical hypothesis or to the smallest number of such hypotheses, it has become customary to suppose that in general a knowledge of the reality is possible and to start out by trying to find sufficient forms to describe the most simple phenomena, reserving the privilege to correct and to generalize as the domain of experience widens.[2]

The openness of mind exhibited in the modern investigations

[2]Cf. *Encyclopédie des Sciences Mathématiques*, Tome 4, Vol. 1, p. 187.

of mechanics is found in many other sciences, and should be more commonly adopted by the teacher who aims to make real progress in the art of teaching. It calls for corrections and reforms on the part of the mathematics teacher, not only on account of the widening of the domain of experience but also, and more largely, because the growth in knowledge of the teacher makes it more and more unnatural to view things from the narrow standpoint from which the immature student is compelled to look at these things. It is especially important that in these days when reform as regards subject matter is so prominently discussed, we should not forget the more serious reforms which are more subjective and hence less easily understood and less easily established.

A LIVING THEOREM.

By Frances B. Hatcher,
Atlanta, Ga.

The following theorem was demonstrated by one of the geometry classes of the North Avenue Presbyterian School of Atlanta, Ga., last May on class day.

The smaller triangle was formed by three girls, each with arms outstretched, shoulder high, representing a side of the triangle. At each vertex where the left hand of one met the right hand of another, was tied a bow of cheesecloth of appropriate shade. The larger triangle was similarly formed except that each side was made up of two girls. The vertices were distinguished by red, blue, and violet bows.

While a member of the class at one side of the platform gave the proposition, the girls of the triangles made the indicated changes in their figures, and also added suggested gestures. For instance, GG′, seemed averse to taking the position required of her. Immediately, the girls in the larger triangle brought their right hands down on their left palms in a most positive and commanding manner, and then as GG′ hastened to obey they brought their arms back to position.

Working out the theorem gave our class a good deal of pleasure, and I hope that it may prove suggestive to other classes.

Theorem: Freshmen should be required to treat all members of the upper classes with due respect.

GIVEN: The equilateral triangle GG′G″ or Green, Greener, Greenest, representing the immature and innocent freshmen of any school, as N. A. P. S., and the similar triangle RBV, composed of superior beings who have long since left childhood's days behind.

To PROVE: That $\triangle$GG′G″ is equilateral and therefore most nearly perfect only when it assumes the proper attitude toward RBV.

PROOF: *Case I.* When $\triangle$GG′G″ is not properly related to RBV.

(1) Suppose that any side of $\triangle$GG′G″, asGG′, feels greater than RB.

Then her head will become so inflated that she will cease to give proper attention to maintaining her position, and will become less than either of the other two sides of her triangle.

(2) Suppose that G′G″ feels equal to RB.

Then, in order to become what she feels herself to be, she loosens her grasp on the other two sides, the angle opposite her becomes a right angle as the two adjacent angles are decreased, and $\triangle$GG′G″ becomes a scalene triangle, with nothing about her right except one angle, and that is wrong in an equilateral triangle.

Case II. When $\triangle$GG′G″ takes the proper attitude toward RBV.

Bisect the side RB at X. Then place GG′ so that G coincides with X and GG′ takes the direction of RV.

If GG′ will not reach the point Y, the midpoint of BV, let fall perpendiculars from each of the sides of $\triangle$RBV, and GG′ will take the proper position.

$\therefore$GG′ now bisects the two sides of $\triangle$RBV, it is parallel to RV, and takes directions from RV or from any other members of upper classes. (Like powers of equal authorities are equal).

In the same way, place GG″ and G′G″ in the positions of XL and LY.

Then the three sides of GG′G″ are equal. (If equals be operated on by positive equals, etc.).

Also, the angles of GG′G″ are now equal for the upper class members composing $\triangle$RBV are now well pleased with the behavior of their younger sisters. (Angles whose "compliments" are equal are equal.)

$\therefore$When $\triangle$GG′G″ shows proper deference to $\triangle$RBV, it remains equilateral and is therefore most useful and happy.

Q. E. D.

THE LAW OF COSINES VERSUS THE LAW OF TANGENTS.

By R. M. Mathews,
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A triangle is uniquely determined by two sides and the included angle. The unknown parts may be found graphically by drawing the triangle to scale and measuring them, or by computation for which trigonometry is necessary, and then three procedures are possible.

All triangles can be solved by reducing them to combinations of right triangles and using the definitions of the trigonometric functions for acute angles. This, however, is a possibility, not a systematic law and method. It is good as a source of problems for bright pupils before the laws applicable to oblique triangles are proved.

All triangles can be solved by the law of sines,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c},$$

together with the law of cosines,

$$a^2 = b^2 + c^2 - 2\,bc \cos A.$$

This fact is explicitly noted in the *Syllabus of Mathematics*, published by the Society for the Promotion of Engineering Education. Yet in not one of fifteen textbooks, that I have examined, is attention called to this fact. Indeed, seven of them do not even note that the law of cosines is of any use in the solution of any triangle; three merely mention the fact that the third side, given two sides and included angle, can be found by said law; and the five others limit its application to cases of arithmetical calculation where the numbers can be squared easily. The last eight books take this attitude because the formula involves *sums* and so *is not amenable to logarithmic calculation.* Wherefore, the law of tangents,

$$\tan \tfrac{1}{2}(B - C) = \frac{b-c}{b+c} \tan \tfrac{1}{2}(B+C),[1]$$

is proved and its use is recommended for the solution of this case.

Recently some pupils of mine experimented by actually using

[1]Why do writers persist in writing the law in the form $\tan \frac{1}{2}(B-C) = \frac{b-c}{b+c} \cot \frac{1}{2} A$, and in using $\cot \frac{1}{2} A$ explicitly as a step in the computation when the other form is symmetrical and $\frac{1}{2}(B+C)$ has to be found anyway to calculate B and C?

logarithms to obtain the squares and square root needed in the law of cosines, and declared the work of solving the triangle no harder by this method than by the law of tangents. This raises the problem: Is there any practical superiority of one method over the other and what is it?

To make a fair comparison between the two methods, each should be reduced to a system of a minimum of entries consistent with accuracy and speed. Inspection of the above-mentioned texts showed that the model solution in only one book was a model of compactness and order. Accordingly, two skeleton forms were arranged as shown below. The following considerations influenced the arrangement of these forms.

Speed is promoted by a form of calculation with a minimum of entries; brevity in those entries; a natural order for making them become habit; compactness to relieve eyestrain in combining numbers; mental work as simple as possible. Speed in the use of tables depends on number of entries; number of times to "take-up" tables; time lost in turning pages; directness from table to page.

Accuracy is promoted by relief in attention afforded by simple mental operations; placing logarithm as near its number as possible; placing logarithms in columns; systematic checks.

Moreover, when a pupil attacks a problem he is helped by having some habitual reactions as a basis. In the present instance he needs (1) to have a definite place to write data; (2) to know how to plan his work, (3) to state definitely the results required. Finally, he will use checks when there is systematic provision for them.

For the skeleton forms here presented, the paper is ruled into four equal columns; each column is devoted to plain numbers alone or to logarithms alone. Paper ruled in horizontal lines is recommended so that one line can be given to each number and its logarithm and the work can proceed by columns, when convenient, with proper spacing assured. In each case column one is to be headed "Measure" and is filled with data or skeleton as shown. Then the complete skeleton is to be made out in the other columns on each of which the heading, "Logarithm," or "Number," is made as shown. Each example is made with the work carried to the stage where logarithms are first needed. Those logarithms marked with a single star are *all* filled in at the one time the tables are first "taken-up"; all antilogarithms

marked with one dagger are "taken-out" together, and similarly for two stars, two daggers, and so forth. These stars and daggers do not belong in the pupil's work, of course.

Solution by Law of Cosines.

Formulas: $a^2 = b + c^2 - 2bc \cos A$, $\sin B = \frac{b \sin A}{a}$, $\sin C = \frac{c \sin A}{a}$.

Measure.	Logarithm.	Logarithm.	Number.
$2 = 2$	$2 = 0.30103$		
$b = 748$	$b =$ *	$b^2 =$	+
$c = 375$	$c =$ *	$c^2 =$	$b^2 =$ †
$A = 63° 35' 30''$	$\cos A =$ *		$c^2 =$ †
	$2bc \cos A =$		—
			$b^2 + c^2 =$
			$2bc \cos A =$ †
$a =$ ††	⟵ $a =$	⟵ $a^2 =$ **	⟵ $a^2 =$
	$1/a =$		
$B =$ ††	$\sin A =$ *		
$C =$ ††			
	$\sin B =$		
$A + B + C =$	$\sin C =$		

Solution by Law of Tangents.

Formulas: $\tan \frac{1}{2}(B - C) = \frac{b - c}{b + c} \tan \frac{1}{2}(B + C)$; $a = \frac{b \sin A}{\sin B} = \frac{c \sin A}{\sin C}$.

Measure.	Logarithm.	Logarithm.	Logarithm.
$b = 684$		$b =$	$c =$ *
$c = 573$			
$b - c = 111$	$- b - c =$ *		
$b + c = 1257$	$- b + c =$ *		
		+	+
$A = 57° 29' 24''$	quotient =	$\sin A =$ *	$\sin A =$
$B + C = 122° 30' 36''$			
	+		
$\frac{1}{2}(B + C) = 61° 15' 18''$	$\tan \frac{1}{2}(B + C) =$ *		
$\frac{1}{2}(B - C) =$ †	⟵ $\tan \frac{1}{2}(B - C) =$	—	
$B =$		$\sin B =$ **	—
$C =$			$\sin C =$ **
$A + B + C =$			
$a =$ ††		$a =$	$a =$

In using the law of cosines: the logarithm of 2 is to be known by heart and considered as a datum; the calculation proceeds horizontally to the right over the first five lines, then back on the next line to the left; with log a obtained, its colog ($\log 1/a$) is written under it; then log sin B, log sin C are computed as indicated by the formulas, and finally a, B, and C are taken from the tables. The proper sign must be provided for $2\,ab \cos A$ according as A is acute or obtuse. In "taking-out"

B and C, the supplement of the tabular value of one may be its true value, so care must be taken to have the largest angle opposite the largest side. The values for these two angles depend on the value found for a, so the sum of all the angles being 180° is a check on the whole work.

The tables are "taken-up" four times; there are five logarithms and six antilogarithms to "take-out," and the two trigonometric logarithms are necessarily on the same page. Total entries, 25.

In using the law of tangents, the computation is in column two to get log tan $\frac{1}{2}(B-C)$; thence $\frac{1}{2}(B-C)$ is "taken-out," B and C computed, log sin B and log sin C "taken-out," log a calculated and a found. The value of B+C is written down by taking A mentally from 179° 59′ 60″. The sum of the angles here being 180° is a check only on the immediate arithmetic of adding and subtracting $\frac{1}{2}(B+C)$ and $\frac{1}{2}(B-C)$. Therefore a should be computed in the two ways shown.

The tables are "taken-up" four times; there are eight logarithms and two antilogarithms, none necessarily on the same page. Total entries, 25, one being a copy.

In all cases where several logarithms are to be taken, time will be saved in turning pages if the numbers be taken in their numerical order.

When these two schedules of essential steps are examined, the comparison shows that *when proper arrangement is made for the work*, the law of cosines is on a par with the law of tangents for solving a triangle, given two sides and the included angle. Furthermore, to find the third side alone, only the checking steps on a can be omitted under the law of tangents, while under the law of cosines the last seven entries can be omitted. The work reduces to: 21 entries, six logarithms and two antilogarithms; and to 18 entries, four logarithms and four antilogarithms, respectively. This provides no check in either case, however.

American textbooks are open to criticism when they nearly unanimously recommend that the side a be found by the law of sines after the angles B and C are found. They make this recommendation because they seldom develop Mollweide's formulas:

$$a=\frac{(b+c)\cos\frac{1}{2}(B+C)}{\cos\frac{1}{2}(B-C)},\quad a=\frac{(b-c)\sin\frac{1}{2}(B+C)}{\sin\frac{1}{2}(B-C)}.$$

The use of these formulas has several advantages. Each formula contains all six parts of the triangle, and consequently, any error in any one of the parts already found is most likely to cause discrepancy between the two values for a. Their principal advantage, however, lies in the way logarithmic work is reduced. We do not need to take out log b or log c; we have log $(b+c)$ and log $(b-c)$ already; and finally, the logarithms of $\cos\frac{1}{2}(B+C)$, $\sin\frac{1}{2}(B+C)$, $\cos\frac{1}{2}(B-C)$, and $\sin\frac{1}{2}(B-C)$ can be taken from the same lines on the tables that give log tan $\frac{1}{2}(B+C)$ and log tan $\frac{1}{2}(B-C)$, respectively.

A skeleton form may be arranged as shown. Here the tables are "taken-up" only three times; there are seven logarithms to "take-out" and two antilogarithms, all on five pages. There are 25 entries, but two of these are copies.

SOLUTION BY LAW OF TANGENTS AND MOLLWEIDE'S FORMULAS.

Formulas: $\tan\frac{1}{2}(B-C)=\dfrac{b-c}{b+c}\tan\frac{1}{2}(B+C)$, $a=\dfrac{(b+c)\cos\frac{1}{2}(B+C)}{\cos\frac{1}{2}(B-C)}$

$$=\frac{(b-c)\sin\frac{1}{2}(B+C)}{\sin\frac{1}{2}(B-C)}$$

Measure.	Logarithm.	Logarithm.	Logarithm.
$b=552$			
$c=328$			
$b-c=224$	$-$ $b-c=$ *		$b-c=$
$b+c=880$	$b+c=$ *		
		$b+c=$	
$A=65°\ 38'\ 36''$	1 quotient $=$		
$B+C=114°21'24''$			
	$+$	$+$	$+$
$\frac{1}{2}(B+C)=57°\ 10'\ 42''$	$\tan\frac{1}{2}(B+C)=$ *	$\cos\frac{1}{2}(B+C)=$ *	$\sin\frac{1}{2}(B+C)=$ *
$\frac{1}{2}(B-C)=$ †	$\leftarrow\tan\frac{1}{2}(B-C)=$	$-$	$-$
$B=$		$\cos\frac{1}{2}(B-C)=$ **	$\sin\frac{1}{2}(B-C)=$ **
$C=$			
$A+B+C=$			
$a=$ ††	$\leftarrow$	$a=$	$a=$

Professor Robert Andrews Millikan, of the Department of Physics in the University of Chicago, has been invited to lecture at the University of Kansas before the local chapter of Sigma Xi, the honorary scientific fraternity. He is the author of a number of works on electricity and magnetism, has received the Comstock prize from the National Academy of Sciences, and is at present engaged in preparing a volume for "The University of Chicago Science Series," to be published under the title of *The Isolation and Measurement of the Electron.*

STUDY TIME OF HIGH SCHOOL PUPILS.

By Marie Gugle,
Supervisor of High Schools, Columbus, Ohio.

This year, when the course of study for grades seven to twelve is being revised, it seemed appropriate to investigate the time spent in preparation of lessons, either in or out of school by pupils of high school grade, in order that any inequalities might be adjusted. The results of this investigation may be of interest to the readers of School Science and Mathematics.

Each high school pupil was asked to list his subjects and give the approximate time, correct to the nearest fifteen minutes, spent in daily preparation. Since the number reporting is so large (almost 3,500), most inaccuracies will be offset; so these statistics are as reliable as any such can be.

These lists were carefully tabulated for each high school, for boys and girls separately. The accompanying table shows the totals for all the schools. The figures refer to the number of pupils spending approximately the time given at the top of the column in daily preparation of the subject listed on the same line at the left. For example, eighty-three pupils spend fifteen minutes daily in preparing first year English; fifty-six pupils spend an hour and a half daily on fourth year Latin. The figures at the right give the total number studying the subject as per reports. There were 3,227 studying English; 1,601, Latin; and 2,187, science.

Forty-five minutes is probably a fair time to devote to the study of a lesson in addition to an equal period of recitation. In English, the numbers center on the 45-minute period with 1,242, but 955 take only thirty minutes and about two-thirds as many, an hour.

Latin centers around the hour, with large numbers spending even more time. In the senior year the center is on an hour and a half, and many listed under the hour and three-quarters or more, really reported two or two and a half hours. In English, out of a total of 3,227, only twenty-two take the maximum time; in Latin, out of half that number (or 1,601), one hundred seventeen take the maximum. For other subjects, compare the figures in the table.

Science and mathematics both center at forty-five minutes; but in each case the number at the half hour is about double that at the hour, which shows that, in these subjects, the tendency is to-

wards a period less than forty-five minutes rather than more; but in Latin and German the tendency is toward a longer period. History is about evenly balanced, with 338 for the half hour and 330 for the hour.

COLUMBUS HIGH SCHOOLS.

Time Spent in Preparation With Number of Pupils for Each 15-Minute Period.

Year.	Subject.	15 min.	30 min.	45 min.	1 hr.	1:15	1:30	1:45 or more.	Totals
1st	English	83	387	467	222	16	40	8	
2nd	English	69	269	358	156	26	28	8	
3rd	English	50	203	262	126	11	17	4	
4th	English	14	96	155	114	11	25	2	
		216	952	1242	618	64	110	22	3227
1st	Latin	25	94	240	231	53	80	21	
2nd	Latin	9	47	113	151	41	59	32	
3rd	Latin	4	9	35	76	23	70	31	
4th	Latin	1	0	10	39	18	56	33	
		39	150	398	497	135	265	117	1601
1st	German	39	139	199	143	18	14	2	
2nd	German	14	64	108	107	19	25	6	
3rd	German	8	43	68	54	17	15	2	
4th	German	2	17	24	26	3	13	4	
		63	263	397	330	57	67	14	1193
3rd	French	7	59	88	41	7	9	1	
4th	French	2	21	36	24	1	6	1	
		9	80	124	65	8	15	2	303
	Spanish	2	15	22	6	0	0	0	45
2nd	History	54	191	367	182	24	23	11	
3rd	History	11	50	101	66	9	24	11	
4th	History and Civics	25	97	171	82	14	33	4	
		90	338	639	330	47	80	26	1550
Physical	Geography	141	407	412	165	9	21	10	
	Botany	34	85	65	17	1	0	0	
	Zoölogy	9	48	46	13	0	0	0	
	Biology	3	4	15	22	5	7	2	
	Physiology	6	15	9	0	0	0	0	
	Agriculture	2	9	5	0	0	0	0	
	Chemistry	20	104	138	59	5	5	6	
	Physics	13	52	115	62	6	12	3	
		228	724	805	338	26	45	21	2187
1st	Algebra	53	285	371	183	30	33	10	
2nd	Geometry	93	202	279	110	19	22	14	
3rd	Geometry	29	80	74	35	6	3	0	
3rd	Algebra	2	20	34	12	3	6	2	
		117	587	758	340	58	64	26	2010

Some one may ask what conclusions may be drawn from these figures and of what use are they. Different persons will draw different conclusions; but one that seems evident is that Latin in all years, and German in some, are demanding too great a

share of the pupils' time. The Latin course has not been changed since the time when it was one of three studies, instead of one of four, and when the pupil had very few demands made upon his time and energy, other than those of his studies. But conditions have changed. Many other activities are now considered a necessary part of his education. In the revised course of study, based on the six-and-six organization, this matter can be adjusted. It is far better to adapt the course in Latin to present conditions, than to have it lose its position entirely.

The one subject that has had to bear the accusation of being the hardest and the one chiefly responsible for pupils' leaving high school, is mathematics. That charge does not seem to be warranted, since this table shows that 1,522 pupils prepare their mathematics in three-quarters of an hour or less, and only 488 spend an hour or more on it.

Since nearly all of the subjects center rightly around the 45-minute period, I believe we can safely conclude that, on the whole, our pupils are not overworked; that is, the amount of work required is fair, and other reasons must be found to explain the occasional breakdown, usually attributed to overwork, such as the strain under which the work is done, unwise division of pupils' time, or indiscretions in diet and social activities.

MUDDIEST OF RIVERS.

The Missouri is the muddiest river in the Mississippi Valley; it carries more silt than any other large river in the United States except possibly the Rio Grande and the Colorado. For every square mile of country drained, it carries downstream 381 tons of dissolved and suspended matter each year. In other words, the river gathers annually from the country that it drains more than 123,000,000 tons of silt and soluble matter, some of which it distributes over the flood plans below to form productive agricultural lands, but most of which finds its way at last to the Gulf of Mexico.

It is by means of data of this kind that geologists compute the rate at which the lands are being eroded away. It has been shown that Missouri River is lowering the surface of the land drained by it at the rate of one foot in 6,036 years. The surface of the United States as a whole is now being worn down at the rate of one foot in 9,120 years. It has been estimated that if this erosive action of the streams of the United States could have been concentrated on the Isthmus of Panama it would have dug in seventy-three days the canal which has just been completed, after ten years' work, with the most powerful appliances yet devised by man.—*Overland Guidebook, Bulletin 612, U. S. Geological Survey.*

The annual statement of the Geological Survey on sand and gravel for 1914 is now available for distribution. During the year 79,281,735 short tons of sand and gravel, having a value of $23,846,999, were produced in the United States.

SOME EXPERIMENTAL RESULTS IN THE TEACHING OF ELEMENTARY SCIENCE.

By G. M. Ruch,

High School, Ashland, Oregon.

Elementary science at the present time in the western states is just beginning its battle for a place in the high school curriculum. The West is fortunate in being able to follow the progress of the East in most educational movements. In Oregon the elementary science movement is but two years old and is confined to relatively few schools.

In the midst of the present controversy over the merits and demerits of such a course, it is evident that actual experimental data is desirable. For this reason elementary science was given a trial in the Ashland, Oregon, high school during the second semester of the year, 1914-15. The data which is presented here represents the results of this trial.

Two facts, it is believed, serve to make this data all the more significant; viz, more than a full year's textbook of material was given in a single semester, together with 18 complete laboratory exercises, and, secondly, the class numbered 43 pupils in one division. The textbook used was Caldwell and Eikenberry's *General Science* but the course was by no means confined to a single book.

Thirty-five of the members of the class were directly from the eighth grade and the remaining eight were all ninth and tenth grade pupils without previous science work.

At the very first period for recitation, without any previous notification, the following questionaire was handed to the class:

I. Why are you taking elementary science?

II. Do you have a definite idea of the scope and nature of this course?

III. When you entered high school did you have your mind made up to take any certain science?

IV. Which science do you now prefer to take?

Forty-two answers were received and were classified as shown on the following page.

It will be noticed that several mentioned more than one subject.

Upon the completion of the course in the following May, a second list of questions was given out to be answered by each pupil. Owing to absences, moving away and sickness, there were

but thirty-three of the class present upon the above date. What effect this condition would have on the results of the test is very problematical. The questions were:

QUESTION I.

Answer given.	Times given.	Percentage of Total Replies.
Teacher or parent's advice	7	16.66
Interest in subject	21	50.00
Help in other studies	5	11.90
Conflicts	6	14.28
To decide what science to take	2	4.77
No answer	1	2.38

QUESTION II.		QUESTION III.		QUESTION IV.	
Yes	19	Yes	20	Physics	6
No	19	No	11	Chemistry	20
Doubtful	4	Undecided	10	Physiography	6
		No answer	1	Biology	5
				No preference	12
				No answer	1

I. Has the work in elementary science caused you to decide to take any other science course during your time in high school? If so, what?

II. What part of the work was the most interesting?

III. Would you like more or less laboratory work than we have done?

IV. Would you prefer to do individual work in the laboratory or have the teacher make all the demonstrations?

V. Write your criticism of this course.

The answers to the questions are given below.

QUESTION I.

Yes	18
No	7
Previously decided	8

Those answering "yes" expressed choices of subjects as follows:

Biology	9
Physics	12
Chemistry	13
Physiography	2
Domestic science	1

QUESTION II.

Five pupils found the topic "Life upon the earth" the most interesting. Four voted for climate; three for air pressure; two each for gases, work and energy, and physiology; three for laboratory experiments; three did not answer; and each of the following topics received one choice—struggle for existence, water pressure, reproduction, steam engines, pulleys, chemical compositions, planets, minerals, and ice plants.

The actual topics enumerated were carefully analyzed according to the science represented as accurately as it was possible to make the determinations. The percentages of this content belonging in each special science were:

	Per Cent.
Physics	26.67
Chemistry	15.00
Physiography, including astronomy	25.00
Biology	33.33

QUESTION III.

More laboratory work	25 answers
Less laboratory work	3 answers
Same amount	5 answers

QUESTION IV.

Number preferring individual laboratory work	23
Number preferring demonstrations by teacher	8
Number undecided	2

QUESTION V.

Thirty pupils answered this question, three omitting it entirely.

Number of answers entirely favorable to the course	17
Number of answers partly favorable to the course	5
Number of answers entirely unfavorable to the course	0
Number of answers favorable but suggesting improvements	8

An examination of the replies to the last question shows that six papers are agreed that "the course gives a good idea of the various sciences." A few of the other criticisms are—"more interesting than I anticipated," "prevents monotony," "too many topics treated," "all right but not enough mechanics and physics," "very practical study," "has made me think of more things," "many of the things come up in everyday life," etc., etc. Many other equally good answers were received but must be omitted for lack of space.

In summary, it seems that several conclusions may be drawn from these two sets of answers, as follows:

1. Elementary science appeals to the pupil just entering high school.

2. Elementary science aids the pupil, materially, in his choice of what other science courses to elect.

3. Elementary science appears to influence pupils to take further work in science.

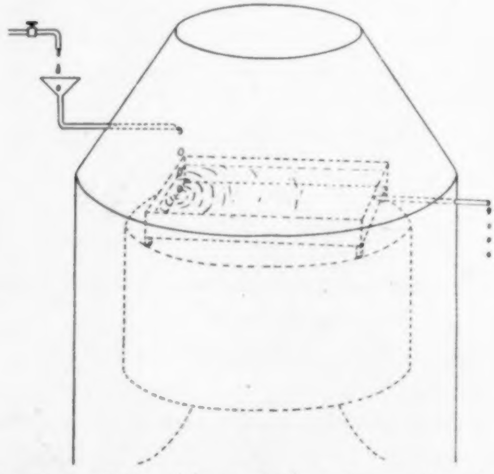
4. The verdict of those pupils who have taken the course is almost unanimously in favor of adopting elementary science into the high school curriculum.

Other conclusions might be drawn from the figures given above but the meagreness of the data makes this more or less uncertain.

INDOOR HUMIDITY.

By Fred D. Barber,
Illinois State Normal University.

Much has been written in recent years about the low indoor humidity prevalent throughout the winter months when artificial heat is required. The evil effects of low indoor humidity, and the consequent rapid evaporation from every moist surface, upon health have frequently been discussed. Regardless of all that may be said in commendation of a dry climate, such as that of southwestern United States, with moderate winter temperatures enabling the residents to live practically out of doors the year around, little can be said in favor of the winter climate of northeastern United States. The change from the outdoor air with its low temperature and high humidity to the indoor air with its high temperature and low humidity, or *vice versa,* is generally conceded to be a trying ordeal. Many suggestions have been offered for controlling the humidity of residences, schoolrooms and public halls. In some instances efforts in this direction have doubtless been more or less successful; it is to be feared that in most cases they have been less successful. It has been the writer's purpose to make a study of the possibilities of controlling indoor humidity of an ordinary furnace heated residence located in central Illinois during a cold spell.



The residence used for this study is a frame structure approximately 22 by 42 feet, two stories, eight rooms, all rooms in daily use. The air supply for the furnace is fresh, outside air and is

ample and the circulation is perfect. The abundance of air supplied the furnace prevents overheating; it is a *warm air* system not a *hot air* system. The humidifier consists of a seamless copper tray or pan, 18 by 36 inches (4.5 square feet of evaporating surface) supported about two inches above the radiator and within the jacket. The water for the humidifier is secured from the city supply and is controlled by means of a needle valve. The humidifier is provided with an overflow pipe. The furnace dampers are controlled by an automatic heat regulator which operates the dampers on a change of two degrees in the temperature of the living rooms. During this test the temperature was maintained at 70° at the warmest part of the living rooms and about 65° at the coolest part.

The following data were taken during the nine days from December 20 to December 28, inclusive, 1914. This was probably the coldest week of last winter in this region.

RECORD OF OUTDOOR AND INDOOR HUMIDITY.

Sunday, December 20, 1914.

Temperature.	Relative Humidity.	7 a. m.	11:30 a. m.	8:30 p. m.
Maximum, 21°	Indoor Relative Humidity	36%	38%	33%
Minimum, 4°	Outdoor Relative Humidity	82%	81%	80%
Average, 12.5°	Average Outdoor Relative Humidity for the Day 81%.			

Calculation.

Absolute humidity of outdoor air at 12.5° and 81% = .710 grains per cu. ft.

An absolute humidity of .710 grains of indoor air at 70° = 9% — Relative Humidity.

Monday, December 21, 1914.

Temperature.	Relative Humidity.	7 a. m.	12 m.	7 p. m.
Maximum, 20°	Indoor Relative Humidity	35%	34%	32%
Minimum, 5°	Outdoor Relative Humidity	94%	72%	80%
Average, 12.5°	Average Relative Humidity for the Day			82%

Calculation.

Absolute humidity of outdoor air at 12.5° and 82% = .719 grains per cu. ft.

An absolute humidity of .719 grains of indoor air at 70° = 9% + relative humidity.

Tuesday, December 22, 1914.

Temperature.	Relative Humidity.	9 a. m.	12 m.	2 p. m.
Maximum, 19°	Indoor Relative Humidity	35%	38%	35%
Minimum, 0°	Outdoor Relative Humidity	77%	64%	58%
Average, 9.5°	Average Relative Humidity for the Day			66%

Calculation.

Absolute humidity of outdoor air at 9.5° and 66% = .500 grains per cu. ft.

An absolute humidity of .500 grains of indoor air at 70° = 6.3% relative humidity.

Wednesday, December 23, 1914.

Temperature.	Relative Humidity.	8:40 a. m.	12:15 p. m.	4:30 p. m.	7 p. m.	9:45 p. m
Maximum, 22°	Indoor Rel. Humid.	34%	33%	33%	34%	32%
Minimum, 5°	Outdoor Rel. Humid.	79%	76%	76%	86%	86%
Average, 13.5°	Average Relative Humidity for the Day					81%

Calculation.

Absolute humidity of outdoor air at 13.5° and 81% = .744 grains per cu. ft.

An absolute humidity of .744 grains at 70° of indoor air = 9.3% relative humidity.

Thursday, December 24, 1914.

Temperature.	Relative Humidity.	9 a. m.	10:30 a. m.	11 p. m.	4 p. m.
Maximum, 30°	Indoor Rel. Humid.	40%	40%	45%	35%
Minimum, 7°	Outdoor Rel. Humid.	98%	95%	92%	82%
Average, 18.5°	Average Relative Humidity for the Day				92%

Calculation.

Absolute humidity of outdoor air at 18.5° and 92% = 1.062 grains per cu. ft.

An absolute humidity of 1.062 grains of indoor air at 70° = 13.3% relative humidity.

Friday, December 25, 1914.

Temperature.	Relative Humidity.	9 a. m.	11 a. m.	1 p. m.	3:30 p. m.	8 p. m.
Maximum, 15°	Indoor Rel. Humid.	38%	35%	35%	31%	28%[1]
Minimum, -1°	Outdoor Rel. Humid.	78%	64%	57%	62%	72%
Average, 7°	Average Relative Humidity for the Day					67%

Calculation.

Absolute humidity of outdoor air at 7° and 67% = .447 grains per cu. ft.

An absolute humidity of indoor air of .447 grains at 70° = 5.6% relative humidity.

Saturday, December 26, 1914.

Temperature.	Relative Humidity.	7 a. m.	11 a. m.	12 m.	2:30 p. m.	4:30 p. m.	6:30 p. m.
Maximum, 14°	Indoor Rel. Hum.	34%	33%	33%	35%	30%	32%
Minimum, -13°	Outdoor Rel. Hum.	74%	54%	53%	64%	68%	72%
Average, .5°	Average Relative Humidity for the Day						64%

Calculation.

Absolute humidity of outdoor air at .5° and 64% = .376 grains per cu. ft.

An absolute humidity of indoor air of .376 grains at 70° = 4% — relative humidity.

Sunday, December 27, 1914.

Temperature.	Relative Humidity.	9 a. m.	11:45 a. m.	4 p. m.	9:30 p. m.
Maximum, 27°	Indoor Relative Humidity	33%	33%	33%	33%
Minimum, 12°	Outdoor Relative Humidity	80%	74%	78%	84%
Average, 19.5°	Average Relative Humidity for the Day				79%

Calculation.

Absolute humidity of outdoor air at 19.5° and 79% = .954 grains per cu. ft.

An absolute humidity of .954 grains of indoor air at 70° = 11.9% relative humidity.

[1]The humidifier ran dry at this point. The drop in temperature necessitated more heat and this produced more rapid evaporation. It was necessary to open the needle valve wider.

Monday, December 28, 1914.

Temperature.	Relative Humidity.	8:30 a. m.	11 a. m.
Maximum, 28°	....Indoor Relative Humidity................	33%	33%
Minimum, 23°	....Outdoor Relative Humidity...............	82%	76%
Average, 25.5°	....Average Relative Humidity for the Day..........		79%

Calculation.

Absolute humidity of outdoor air at 25.5° and 79% = 1.253 grains per cu. ft.

An absolute humidity of 1.253 grains of indoor air at 70% = 15.7% relative humidity.

Additional Facts.

Average outdoor temperature for the period = 13.3°.
Average outdoor relative humidity (daytime only) = 77%.
Average indoor relative humidity (daytime only) = 34.5% with maximum of 45% and minimum of 28%.

The windows on the west and north side were provided with storm windows and did not frost over; the single windows were covered with frost and much ice accumulated at the bottom of each. The outside wall of one west room showed considerable moisture on Saturday, December 26th.

On Saturday, December 26th, about six gallons of water were evaporated by the humidifier. On the average, probably about four gallons of water were evaporated daily. The water in the humidifier boiled on several occasions and stood near the boiling point frequently.

While the data given cover a period of nine days only, the humidifier has been in use for several years and it is believed that these data indicate correctly the usual operation of the plant when low temperature prevails.

Conclusions.

1. This device is nearly automatic in its operation and does control the humidity of indoor air, maintaining a fairly constant relative humidity of about 35 per cent during both moderate and extreme outdoor temperatures.

2. It is folly to talk of materially increasing the indoor humidity of a well-ventilated residence by using the common furnace water pan or even by suspending vessels of water within registers. Any device which does not evaporate several gallons of water daily in a well-ventilated residence is nearly useless.

3. It is not practical to attempt to maintain a relative humidity of more than about 35 per cent or 40 per cent during cold weather. A higher humidity, if obtainable, results in excessive frosting of windows and the "sweating" of walls during zero weather.

4. While granting that the *calculated* relative humidity given above is considerably lower than actually would have prevailed but for the humidifier, it still is true that the thirsty air would have stolen the additional moisture from every available source such as the building itself, the furniture and the skin and mucous membrane of the residents.

5. A temperature of 65° to 68° in the living room or library is entirely adequate for most people if the relative humidity is maintained at 35 per cent or more.

The records of the Weather Bureau show that the average temperature for January, for eastern Iowa, southern Wisconsin, the Lower Michigan Peninsula, Northern Illinois, Indiana, Ohio, Pennsylvania, New York, New Jersey, and most of New England is from 20° to 30° while the average relative humidity of this region for January is from 75 per cent to 85 per cent. In this region live nearly one-half of the population of the United States and here is found more than one-half of the wealth and resources of the nation. If the control of indoor climate during the winter months is as important for human health and vitality as recent writers assert, we certainly need more information concerning the methods by which it may be obtained.

An outdoor temperature of 25° with a relative humidity of 80 per cent means an absolute humidity of 1.241 grains of moisture per cubic foot. If this air is admitted to the residence, school or counting room heated to 70° without the addition of moisture, a relative humidity of 15.5 per cent results. This is a much lower relative humidity than is to be found in outdoor air in any inhabitable region on the face of the earth.

The effect of frequently passing from indoor air at a temperature of 70° and a humidity of 20 per cent or lower into outdoor air at 25° and 80 per cent humidity is a matter which should receive more attention. If indoor air at lower temperatures and higher humidity than usually found is more conducive to health and vitality, we need more light upon the best methods of obtaining it.

RECENT DEVELOPMENTS IN OUT-OF-DOOR ILLUMINATION.[1]

By Harold Blair,

Shortridge High School, Indianapolis, Ind.

I shall first briefly discuss certain theoretical laws which underlie the operation of any electric light; and apply these laws to the leading types of lamps; secondly, I shall show how the different lamps are adapted to the various classes of out-of-door installations; and, finally, I shall describe the new lighting system of the city of Indianapolis which embodies many of the points discussed under the first two heads.

There are two general methods for the production of light—namely, by incandescence and by luminescence. In the first method, the temperature of the body is raised through the expenditure of energy in overcoming the resistance of that part of the electrical circuit, and this loss of energy appears in the form of heat accompanied by the magnetic and electrostatic field, but no chemical change is apparent or at least progresses very slowly. When the light is accompanied by molecular or chemical changes, we call it luminescence. In the first class is the carbon filament, tungsten, Nernst and ordinary arc lamps (either open or enclosed) while to the second class belong the flaming arc and luminous arc.

When a current is passed through a conductor, there is a loss of energy equal to I^2R, and this energy is given off in the form of ether waves, a part of which are of short enough length to be visible to the eye, while by far the greater proportion are the longer heat waves. If we increase the current through this conductor, the energy radiated becomes greater according to the above formula and the conductor changes from a dull red to a brilliant white heat, and with the change of temperature, a greater and greater proportion of the energy is radiated as visible light of shorter wave lengths. This does not mean that as the temperature is raised, the energy radiated in the red portion of the spectrum diminishes. As a matter of fact it increases rapidly, but the rate of increase is far more rapid in the shorter wave lengths of the spectrum. The whole phenomena follows the displacement law of Wein which states that the product of the wave length at which the radiation is a maximum, Lm, and the

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[1] Read before the Indiana Association of Science and Mathematics Teachers, Indianapolis, March 6, 1915.

absolute temperature is a constant or Lmt = K. If Lm is expressed in m. m., the value of K is 2.92. The commercial efficiency of a lamp is given in watts per c. p. but if we consider its luminous efficiency which is defined as the ratio of the watts output to watts input we can show that the efficiency of a lamp at the best does not approach to the efficiencies that we require in other electrical machinery. Consider an ordinary tungsten lamp with a commercial efficiency of 1.25 watts per c. p. The luminous energy radiated per second by a light of one c. p. has been estimated to be about $1.3x10^6$ ergs. The efficiency of the tungsten is then $\frac{1.3x10^6}{1.25x10^7} = 10.4$ per cent.

The entire problem then, in reaching to a higher efficiency in a lamp which depends on incandescence for the source of the light, is to secure a higher temperature. This was done in the carbon filament lamp by metallizing the filament in the so-called gem lamp, by the introduction of the tungsten lamp, and finally the latest step in the development came last year when the nitrogen-filled tungsten appeared on the market. This last discovery will probably revolutionize practice in outdoor lighting, and of that I shall have more to say later.

If we increase the voltage applied to a carbon filament lamp the light becomes whiter and the efficiency increases rapidly. At say 150 or 160 volts the ordinary 110-volt lamp is emitting a most brilliant white light at an efficiency of possibly .6 or .7 watts per c. p., but after a very short time the interior of the bulb blackens and the filament breaks. The same experiment with a tungsten lamp gives the same results. A tungsten can be run at an efficiency of .2 watts per c. p. for a few seconds but any increase beyond this point results in the filament melting. At .5 or .6 watts per c. p. the bulb blackens in a few hours. In the General Electric research laboratory extensive experiments were made to discover the cause of this blackening of the bulb. It was found that water vapor attacked the tungsten forming hydrogen and a volatile exide of tungsten which deposited on the glass. The oxide later reduced to metallic tungsten with the reformation of water vapor. The blackening of the bulb, however, went on in the entire absence of water vapor. A number of theories were advanced to explain this, chief among which was that there was an emission of electrons from the filament. After much careful research, Dr. Langmuir decided that the cause was nothing more startling than a simple heat evaporation

of the tungsten from the surface of the filament. This can be obviated by the use of nitrogen gas at atmospheric pressure, or else by localizing the black deposit by convection currents. It was proved that the heat loss by convection increases approximately as the 3/2 power of the temperature while the radiated energy increases as the 4th power of the absolute temperature. The present nitrogen-filled tungsten runs several hundred degrees hotter than the old form. Thus the loss due to convection in the nitrogen becomes *relatively* low at high temperatures. It was further shown that the heat loss by convection is about the same in a wire of small diameter at a high temperature, as the loss from a large wire. Therefore the most efficient types of the new tungstens are large low voltage units taking a current of from 20 to 30 amperes and operating on series A. C. circuits. These large units have an efficiency of about .5 or .6 watts per c. p. There are two other types of the nitrogen-filled lamps than the above, viz., a low voltage lamp running on from five to ten volts and taking ten amperes or less, used in automobile headlights or in work of that sort and sometimes used on 6.6 amperes series street light circuits. The efficiency of the smaller units runs from .7 to 1.25 watt per c. p. The third type of these lamps is that used on the standard 110-volt circuits. In these as in the other types the efficiency runs higher in the larger units. In large units of from 1,000 to 2,000 c. p., the efficiency runs .5 watt per c. p. or even better. The light of the nitrogen-filled tungsten is the nearest approach to daylight of any lamp except that of the Macfarlane Moore tube filled with CO_2. It was found that this improvement could not be used in a carbon filament lamp owing to the formation of a carbon nitride. A fine feature of this new lamp is that it runs at its rated candle power with but a very slight loss, and the life of the lamp is ordinarily terminated by the breaking of the filament. In the early form of the tungsten the blackening of the filament usually ended the life of the lamp. (In life tests of lamps, a lamp is "officially dead," when its candle power falls to per cent of its rated value.) At least one prominent manufacturer of these lamps guarantees them to last 1,000 hours.

Luminous Arcs.

In the luminous arcs we have a more direct conversion of electrical energy into light through the luminescence of the material of the arc. The arc conductor between the electrodes is a

metallic vapor at a considerably lower temperature (about 2000°), than the electrodes of the old arc lamp. The electrode is a large metallic electrode usually of copper although the material is not essential. This electrode is placed as the top electrode of the arc in order that its size may check the ascending metallic vapor from the negative electrode and render the operation of the lamp more steady. The negative electrode consists of a carefully welded cylinder of sheet iron of a considerably smaller diameter than the positive electrode. This cylinder is filled with a mixture of chromite, FeO_1 Cr_2 O_3, magnetic oxide of iron FeO, Fe_2 O_3, and the dioxide of titanium. Sometimes the titanium is combined chemically with the iron in order to give a more uniform arc. The negative electrode will burn from 100 to 200 hours without trimming while the positive electrode has a life of from 3,000 to 4,000 hours.

Because of the low temperature of the arc the lamp must not be subjected to any variations of arc length which would result in great variations of the candle power. To operate the lamp in multiple, of course, a ballast must be connected in series with it to render its operation stable, but any attempt to regulate the voltage by varying the distance apart of the electrodes results in great unsteadiness in the volume of the light. The lamp, therefore, operates best on a series circuit where the variations in the voltage of the different lamps overlap, and tend to equalize each other. The circuits for these lamps are run from constant current transformers and mercury arc rectifiers to give a direct current. Since the power factor of the primary of a constant current transformer is rather low, this constitutes a distinct disadvantage, especially when the transformers are operated from the station bus-bars as is usually the case. The efficiency of the luminous arc is from .4 to .5 watt per c. p. and the light is a fine white light giving fairly close approach to daylight. The nitrogen tungstens, however, will probably soon replace the luminous arcs in out-of-door series circuits.

Flame Arc.

The flame arc, unlike the luminous arc, is a carbon arc with the arc stream colored by the heat evaporation of the metallic salts of the electrodes. In the early form of the flame arc, the carbons were cored and the light-giving salt introduced into the core. With this form, the arc would often maintain itself on the edge of the carbon shell and give an inefficient bluish light.

At present the carbons are made by pressing a fine uniform paste containing the ingredients through steel dies, and afterwards baking them to drive off the moisture and to harden them. Calcium fluoride is used as the chief light-giving salt in the yellow flame carbons, while certain rare earths (titanium oxide is one of them) are combined in the white flame carbons. The yellow carbons are used chiefly for display purposes or in special work where the distortion of color is not a disadvantage. At close range the yellow flame carbons give a light which overpowers that given by tungstens or by ordinary arcs, but when seen from a considerable distance they appear dim and smoky beside their whiter neighbors. The yellow flame carbons are more efficient than the white flame carbons but recently there has been a considerable improvement in the latter so that the white flame arcs have come into a wide use for street lighting.

The flame arc is at its best in large units because the carbons must be large enough to facilitate the evaporation of the light-giving salt, and they will operate well on either series or multiple circuits, D. C. or A. C. and with any frequency from 25 to 140 cycles. Because of this necessity for large size the flame arc is not as suitable for low intensity illumination in areas having a low population density, as the old enclosed arc, or the nitrogen-filled tungsten. A common practice in recent work is to run them on series A. C. circuits of 60 cycles on 7½ or 10 amperes. The enclosed flame arc will operate about 100 hours for each trimming and requires 60 volts for its maintenance. It is usual to run the series flame arcs from constant current transformer circuits. An advantage that the flame arc enjoys over the luminous arc is that the latter is a D. C. arc and must be operated from mercury rectifiers. This adds to the cost of the station equipment and maintenance. The lamp gives off fumes which must be condensed in some sort of a condensing chamber in order to prevent them from rendering the globe dim. In the Westinghouse flame arc the condenser is at the base of the chamber containing the mechanism, and the white ash furnishes a white reflecting surface above the carbons. In spite of all precautions, however, some of the fumes will condense on the inner globe, and it must be cleaned thoroughly at each trimming. In time the globe becomes so etched with the deposit that it transmits only 55 per cent of the light.

The enclosed flame arc must be trimmed about every 100

hours, and the stub of the old upper carbon is used for the new lower carbons.

The efficiency of a flame arc is .3 to .4 watts per mean spherical c. p. When the cost of trimming and carbon renewals, together with the initial cost of the lamp is considered, it seems probable that the flame arc also will soon be replaced by the new nitrogen tungstens. In fact, the city of Chicago is putting in all new work with the nitrogen lamps, and the engineering department there is now considering the replacing of many of their present flame arcs with the tungstens. A further improvement in the condensing apparatus of a flame arc, together with a greater efficiency in the white flame carbons, may, however, give it a renewed ascendency.

To summarize: There are at present three types of out-of-door electrical illuminants—the others being regarded as obsolete—the luminous arc, the flame arc and the nitrogen-filled tungsten. Recent developments in the efficiency of the white flame carbons have given it an advantage over the luminous arc, so that we practically have the flame carbon arc and the nitrogen tungstens in a close contest for first place, with the odds in favor of the tungsten.

The new arc lamps of Indianapolis are 10 ampere Westinghouse enclosed flame arcs which operate on 60-cycle A. C. current. The voltage per lamp is about 60. White flame burning about 100 hours will be used. The masts on which the lamps hang are stationary and for trimming they are lowered by means of a rope passing over a pulley attached to the top of the cut-out. When the lamp is lowered it is automatically disconnected from the circuit, thus lessening the danger to the laborer. The circuits are operated on constant current transformers, each capable of carrying 100 lamps. The power factor of the transformers is 65 per cent on the primary and 70 per cent on the secondary side. They are operated at a primary voltage of 2200 from the station bus-bars.

All the boulevard lighting is done by nitrogen tungsten lamps operating on 10-ampere, 60-cycle A. C. circuits. The lamps designed for corners are 170-watt, 250-c. p. rating. The manufacturer guarantees these lamps for 1,000 hours, and all lamps not meeting this requirement are replaced. The voltage across the large lamps is about 17 volts, while that across the smaller units is 11 volts.

These circuits like the flame arc circuits are operated from constant current transformers of precisely the same design and rating as those for the arc circuits. The power factor of the primary on the tungsten circuits is about 80 per cent, and on secondary about 98 per cent.

Film cut-outs are used to keep the other lamps of the circuit burning in case one should burn out.

The design of the posts is plain, but highly pleasing, and the entire installation is thoroughly efficient and a great ornament to the city.

AGATE AND ONYX.

The distinction between agate and onyx is not apparent to everyone, as is indicated by the samples of the two minerals received by the United States Geological Survey with requests for information. Onyx marble, or Mexican onyx, is composed of calcium carbonate or banded limestone. True agate is a variety of silica. Onyx marble is much softer than agate and is rarely used for gems, but when onyx is obtained in pieces of sufficient size it is cut and polished for small ornamental objects like inkstands and paper weights, as well as for table tops and soda-water fountains.

POTASH IN SOUTHEASTERN CALIFORNIA.

The salt-incrusted valley floor commonly known as Searles Lake, in southeastern California, has lately come into prominence through the widespread interest in the search for an available source of potash in this country and the apparently promising prospects this locality affords of a considerable commercial production in the near future. The estimate made three years ago that this deposit contains 4,000,000 tons of water-soluble potash salts seems to have been amply confirmed by subsequent developments. That this amount of potash salts will actually be produced and placed on the market can not yet be considered assured, but so far as can be judged from evidence available, it seems that this deposit is the most promising immediate source of commercial potash in the United States.

In *Bulletin 580-L*, recently published by the United States Geological Survey, Hoyt S. Gale gives a general account of the main features of the lake history and the deposition of the salines through the drying up of the lake waters. His report is a preliminary review based on trips made through this and other parts of the Great Basin in pursuit of the general plan of investigations looking toward the discovery of future potash supplies.

Waters that formerly filled Owens Valley until they overflowed, flooding successively lower and lower basins, formed for a time a chain of large lakes in what is now the desert region of southeastern California. These flood waters passed from Owens Valley, the principal source of the water supply, through Indian Wells, Searles, and Panamint Valleys, in each of which there was an extensive lake. Finally the waters are believed to have overflowed also into Death Valley.

A copy of *Bulletin 580-L* may be obtained free on application to the Director of the Geological Survey, Washington, D. C.

CONDITIONS FOR A HIGH SCHOOL COURSE IN ELECTRICITY.

By C. L. Vestal,

Carl Schurz High School, Chicago.

This paper proposes to discuss the proper conditions for a course in electricity in a technical high school. The first step is to inquire into what should be the scope of such a course.

It would have a two-fold purpose—character discipline, and the acquiring of commercially valuable information, and skill in using same. The first is an accompaniment of the process of acquiring the latter, for if there is any one branch of scientific study which requires concentration and application, it is electricity, practical and theoretical.

The pedagogical and cultural value are linked together in still another way. It is my observation that the value of a student's work to him is largely determined by his perception of its reality, of its genuine significance for life. In the case of electricity that subject is everywhere around him, especially in the city. Its applications and its utility are increasingly ubiquitous. It appeals to the boy, and pretty much also to the girl, as significant as well as merely interesting. It has thus all the qualifications of a desirable part of a curriculum from a pedagogical point of view, in addition to being of growing, and even now great, vocational value. Its place is therefore established.

A technical course in electricity is primarily a course in electrical testing of commercial apparatus and machinery, by means of measurement. In no other technical work is it more true that knowledge is chiefly acquired by measurement. This assumes, of course, that the fundamentals must be mastered before the larger work begins, meaning by "fundamentals" the definitions and conceptions of the units of current, pressure, resistance and power. The other units—capacity, inductance, reluctance, reactance, impedance, and all the phenomena associated with a varying current—should not be taken up until the student is thorough in understanding and skillful in the use of the four fundamental units for steady currents. This understanding and skill must be given by much mental study and class drill as well as by ample laboratory practice, some of it self-directed as well as instructor-directed. The first stones in this excellent foundation will have been laid in the course in physics, for it seems to me that no student who desires the best results from such a course should

enter upon his two periods a day in the electrical laboratory-shop until the beginning of his third year, and should therefore be permitted to take the physics in his second year.

Such a course is in no sense a trade course for apprentices. Apprenticeship should be left to the labor organizations. It does not propose to train the boy to be a wireman, a telephone "trouble shooter," a low-paid armature winder, an assembler of machines, nor any other of the special phases of the work of the man who does the manual labor in the use of electricity. It proposes to make him thoroughly acquainted with the operating characteristics of all commercial electrical machinery, so that he can deal intelligently with any of them, and work easily into the position of supervising engineer.

The value and objects of such a course being as indicated, it follows that the equipment of a department to give it must be with commercial electrical machines.

Commercial electrical apparatus may be, for convenience in discussion, divided into three classes, according to purpose, viz, generation, transmission, and use—the first being simply the changing of mechanical energy into electrical, and a large part of the last is simply changing it back again, electricity thus being the best opportunity which Nature offers to obtain the maximum of flexibility in the use of energy.

Under the head of generation come generators or "dynamos," for both direct and alternating currents. The latter may be subdivided into revolving armature and revolving field machines. Under this head should perhaps also be included the rotary converter, which though not strictly a generator, often takes the place of one.

Under transmission may be listed switchboards, and all switching apparatus, as well as transformers and the actual lines themselves. Owing to the difference in the problems involved, transmission may be subdivided into high tension and low tension. Under the head of transmission should also be included measuring instruments, since these are usually placed at the centers of distribution, in order to show the electrical conditions on any or all parts of the system at any time.

Under use come the subdivisions of heat, light, and power. The first of these includes such devices as flatirons, toasters, percolators, curling irons, soldering irons, water heaters, heating pans, hot plates, and furnaces, both arc and resistance. Under light come the ordinary incandescent lamp, both carbon and

tungsten; the mercury vapor lamp; the Nernst lamp; the new nitrogen-tungsten lamp; all forms of the arc. Under power come all kinds of devices for changing the magnetic accompaniment of a current into motion. This is usually an electric motor, but is sometimes a tractive magnet. The latter are commonly some form of solenoid, and are usually designed for continuous current. Motors may conveniently be subdivided into those for continuous and those for alternating currents. The continuous current motors may be further subdivided in series, shunt, and occasionally some form of compounding. It should be said that the series motors are also often constructed for alternating currents. The shunt motors might be still further classified as to method of speed control or none. Alternating current motors may be subdivided into series, synchronous, and induction. The induction motors are still further differentiated into squirrel-cage and slip-ring types. To this should be added the two well known and widely used types of single phase motors, the split-phase and the repulsion. The study of motors will naturally include the various methods of motor control—the several types of rheostats, adjustable voltage transformers, etc.

The electrical department which claims to offer adequate and thorough training in electricity must be equipped with several pieces of each the classes of machines mentioned in the above sketch. Mere talk about the performance of a piece of machinery is of little value without the observance of the actual performance itself under genuine load and service conditions.

The testing work and lecture demonstration will require both continuous and alternating currents at various voltages. The commercial voltages now in use, barring those for transmission purposes purely, which are too high to be used by the amateur experimenter, are 110-115, 220-230, and 550-600, the last being chiefly direct current for street railway use. The two former voltages are the ones most frequently met with in use, the first for lighting and small power and heating devices, the second for the larger power purposes, in motors of from 1 h. p. to 100 h. p. The department must therefore be supplied with both these voltages in both continuous and alternating currents, and with the necessary transformers and batteries to obtain other voltages when necessary.

Since in the large city most of the service is A. C., we will assume that the school is supplied with that only. This will be generally true except in those schools which maintain their own

power plants. In the large cities the commercial service is preferable. Assuming, then, that the commercial service is A. C., it will be necessary to equip the department with a converting plant of sufficient capacity to run a number of D. C. motors at once, said motors ranging from 1-8 h. p. to 5 h. p., the very small ones using 110 volts, and the larger ones 220 volts. Both these voltages will therefore have to be provided for, in D. C.

This may be done in several ways. The simplest, although perhaps not the cheapest, way is to have two D. C. generators, each wound for 125 volts, direct connected to a single 3-phase 220-volt slip-ring induction motor. The two generators can then be connected in series to one panel of the main switchboard, and each of them also connected to another separate panel. Energy from one panel will therefore be at twice the voltage from either of the other D. C. panels. While simple, this method makes the regulation of the higher voltage rather difficult if the two generators are unequally loaded, as will usually be the case. However, voltage regulation can be improved by a booster set. Still another method is to use a single three-wire generator, with the necessary auxiliaries. Still another is to use two separately driven generators, one of each voltage. Another is to use two rotary converters, and yet another is to use one converter and one generator. From a pedagogical point of view the last has some points in its favor.

But whatever method is used, there should be available at any time at least 30 kw. of energy of each voltage, assuming a class of 24 students at one time. At the main switchboard there will also be available for distribution about the department alternating current of either of the working voltages and of one, two, or three phase. The switchboard itself should preferably be of black slate, and should be equipped with all the necessary instruments for making observations of any or all of the quantities involved at any time—volts, amperes, watts, watthours, power factor, frequency, and speed of rotation. Each panel will also, of course, be equipped with a circuit breaker. This will mean eight panels in the main board. Not many lever switches will be used. In the main laboratory should be eight experiment tables, each 2′x7′, and 30″ high, open underneath, but with six drawers in each end. Each table will be provided with outlets, mounted on slabs of slate, so that any current of any voltage or phase may be furnished thereto.

The machines for testing will be mounted on tables 30″ high

and 2′ wide, and of whatever length is necessary. These tables will have very thick legs, because of the heavy weight they must carry, and will be fastened to the floor with angle irons. The floor itself should be of wood, well oiled. There should be provided two D. C. motor-generator sets and two A. C. sets, so arranged with their own switchboards that there may be exercises of connecting them to work in parallel. There should be at least three other D. C. motor-generator sets, to permit a considerable proportion of the class to study the characteristics of generators at once, or to use the motors to test. There should be at least two or three free motors of every type, from $\frac{1}{8}$ to 5 h. p., but most will be from 1 to 3 h. p. The larger ones should all be mounted on tables as described. The same is true of the several transformers.

It should be said here that one of the first steps toward the establishment of a successful department is to get rid of the idea that one room is enough. It is not. There should in this case be a main testing laboratory, which should not contain less than 3,500 square feet of floor area. Then there should open from it an adequate lecture room, amphitheater style, seating about forty students, and adequately equipped. Opening back of the lecture table should be a small, tight room for lecture-table apparatus. This room need not be more than 10′x20′. Also opening off the main testing laboratory there should be a long, narrow dark room for measurements on lamps. This room should be about 10′x30′, and should also contain the wall galvanometers, and the standardizing instruments—potentiometers, condensers, standard cells, resistance-boxes of guaranteed accuracy, etc. Opening off this room it would be convenient, though not essential, to have a small photographic developing room, about 5′x7′, for the use of the instructor in making lantern slides.

The equipment of the main laboratory will also include several construction machines—two small armature winders, a punch press with the proper assortment and size of dies, and a large finely adjustable lathe. This is because a considerable part of the course, at the latter end, will consist in applying the rules of design, in requiring the student to actually design and construct one machine of each kind: motor, transformer, speed-control rheostat.

In another small separate room, but connecting to the main switchboard, will be a large storage battery, with all the proper

accessories for ventilation and watering. The battery will connect with the outlets in the room from the main switchboard.

Also opening from the main laboratory, and separated from it by a partially glazed partition, should be a small room to be used as an office for the instructor and as a library for reference by the students. It should be about 12'x16', and should contain two reading tables, and be equipped with a small, but comprehensive working library on the subject.

An apparatus and supply room should also open off the main laboratory. This should be about 12'x24', and will contain the necessary cases for portable apparatus and general supplies.

In the main laboratory one wall space, the full ceiling height and about 25' long should be covered with soft wood and reserved for wiring practice.

Now of course the immediate and perhaps somewhat clamorous objection growing in your mind will be to the great expense of such an arrangement. Expensive it undeniably would be. The proper equipment alone plus the cost of installation for such a department would probably not be less than $40,000 to $45,000. However, sums approximating this have been spent for technical shop equipment in single installations in smaller cities than Chicago. The mere machine-shop equipment of a technical school comes to a pretty figure, and this has been spent without a murmur. Yet its work is no more valuable, either culturally or commercially, than this would be. As an offset to its great initial cost, it should be remembered that such an equipment is good for a generation of service, if properly cared for. Assuming classes of twenty-four students each, suppose that only two such classes completed the work each semester, this would serve to equip with its preparation nearly 3,000 students before much replacement would become necessary. Also it would be exceedingly popular with evening school students, to probably an even greater number than the day school pupils.

Also such a department would offer work of a grade two years beyond the regular four year high school course. It is not too much to say that it could well take the place of the electrical engineering colleges for a large number of students. In fact the practical testing work which it would offer would probably be superior to that offered in the electrical engineering courses of many colleges. On the whole, I am convinced that the establishment of such a department on an adequate basis would be an excellent social investment.

RULE FOR EXTRACTING THE n-th ROOT OF ARITHMETICAL NUMBERS.

By William T. Short,

Oklahoma Baptist University, Shawnee, Oklahoma.

1. Separate the number into periods of n figures each.
2. Find the largest nth root of the first period.
3. Raise the root to the required power and subtract.
4. Bring down the next period.
5. Add a naught to the figure (or figures) already found and raise to the $(n-1)$th power.
6. Multiply this by the index and use as a trial divisor. (Or, in place of multiplying, divide the remainder by this number and the quotient by n.)
7. Place the number thus found as the next figure of the root and also use as directed in operations 8 and 9.
8. In operation 5 replace one of the naughts by the new figure of the root and multiply.
9. Repeat till all the naughts are exhausted.
10. Add the products thus found and multiply their sum by the figure of the root last found.
11. If this product is smaller than the remainder, subtract and start anew with operation 5.
12. If this product is larger than the remainder the last figure of the root must be decreased and operation 7 and following repeated.

The following example will illustrate the rule.

Find the fifth root of 33554432.

			355 · 54432	32
	$3\times3\times3\times3\times3$	$=$	243	
$925\div81 = 11+$,	$30\times30\times30\times30$	$=$	810000	92 54432
$11\div5 = 2+$.	$30\times30\times30\times32$	$=$	864000	
2 is the second	$30\times30\times32\times32$	$=$	921600	
figure of the root.	$30\times32\times32\times32$	$=$	983040	
	$32\times32\times32\times32$	$=$	1048576	
			4627216	92 54432

Find the fourth root of 152587890625.

			1525 · 8789 · 0625	625
	$6\times6\times6\times6$	$=$	1296	
$2398\div216 = 11+$,	$60\times60\times60$	$=$	216000	229 8789
$11\div4 = 2+$.	$60\times60\times62$	$=$	223200	
2 is the second	$60\times62\times62$	$=$	230640	
figure of the root.	$62\times62\times62$	$=$	238328	
			908168	181 6336
	$620\times620\times620$	$=$	238328000	$=$ 48 2453 0625
$4824\div238 = 20+$,	$620\times620\times625$	$=$	240250000	
$20\div4 = 5$.	$620\times625\times625$	$=$	242187500	
5 is the third	$625\times625\times625$	$=$	244140625	
figure of the root.			964906125	48 2453 0625

A CORRECTION.

Near the bottom of Page 487 of the June, 1915, issue of this Journal, the statement is made that the National Wool Warehouse and Storage Company, Chicago, will furnish free exhibits of wool, yarn, and cloth. This is a mistake, as this company is unable to supply these exhibits free of all cost.

PROBLEM DEPARTMENT.

BY J. O. HASSLER.
Englewood High School, Chicago.

Readers of this magazine are invited to propose problems and send solutions of problems in which they are interested. Problems and solutions will be credited to their authors. Address all communications to J. O. Hassler, 2301 W. 110th Place, Chicago.

Foreword.

The new editor takes this opportunity to thank the contributors to this department for the felicitations they have sent and for the hearty support that is being given the new management. The worthy and efficient manner in which the former editors of the department have conducted it places a great burden of responsibility on one who is very slightly acquainted with this kind of work. He desires to make the material in this section of maximum interest and value to those who read it. To that end he wishes the contributors to send any suggestions they feel prompted to make which would improve the department.

Problem 444 suggests to the editor a line of work both profitable and interesting, namely, the discovery of fallacies. Mathematicians must always be on guard against errors. We find them even in the work of the master minds. To train us in a small way along this line we might have, regularly, fallacious solutions to correct. Let each contributor express his opinion when he sends in his next solution and the editor will be guided by the wishes of the contributors. Send along the fallacies, too.

Algebra.

441. *Proposed by Herbert N. Carleton, West Newbury, Mass.*

A man can row y miles per hour on a certain river with the current in his favor, but only x miles per hour against the current. The numbers x and y are such that $x = \sqrt[x]{y} = y^{\frac{x}{y}}$. Starting from point A, rowing down stream as far as point B, and returning to his starting point, rowing continuously at the rates above specified, he makes the round trip in $7\frac{1}{2}$ hours. Find the distance AB, numerically expressed.

I. *Solution by R. M. Mathews, Riverside, California, and Fenton V. Stearns, Westerville, Ohio.*

From $x = \sqrt[x]{y}$, $x^x = y$, and from $\sqrt[x]{y} = y^{\frac{x}{y}}$, $\frac{1}{x} = \frac{x}{y}$ and $y = x^2$.

$\therefore \quad x^x = x^2$, whence $x = 2$ *and* $\therefore y = 4$.

Let d denote the distance AB. Then

$\frac{d}{2} + \frac{d}{4} = \frac{15}{2}$, whence $d = 10$.

II. *Solution by Edward Fleischer, Bushwick High School, Brooklyn, N. Y.*

$$x = y^{\frac{1}{x}} = y^{\frac{x}{y}}$$

$$\log x = \frac{1}{x}\log y = \frac{x}{y}\log y.$$

Clearing of fractions and reducing we get

$$xy \log x = x^2 \log y \qquad (1)$$

$$y \log y = x^2 \log y \qquad (2)$$

From (2) $\quad y = x^2$, whence, substituting in (1),

$$x^3 \log x = x \log x^2,$$

or $\quad x^3 \log x = 2x \log x,$

whence $\quad x = 2.$

$\therefore \quad y = 4.$

Let z be the distance required.

$$\frac{z}{2}+\frac{z}{4}=\frac{15}{2}, \text{ whence } z = 10.$$

III. *A mental solution by T. M. Blakeslee, Ames, Iowa.*

$$x = y^{\frac{1}{x}} = y^{\frac{x}{y}}. \quad \therefore \frac{1}{x} = \frac{x}{y}, \; y = x^2, \; x = x^{\frac{2}{x}}.$$

$$\therefore \; \frac{2}{x} = 1, \; x = 2, \; y = 4.$$

To go and return one mile takes $\frac{1}{2}+\frac{1}{4} = \frac{3}{4}$ hours. In $\frac{15}{2}$ hours one can go and return $\frac{15}{2}\cdot\frac{4}{3} = 10$ miles. All of this is easily done in the mind.

Geometry.

442. *Proposed by Mabel G. Burdick, Stapleton, N. Y.*

The locus of points whose distances from two fixed points have a given ratio $(m : n)$ is the circumference of a circle whose diameter divides the line joining the fixed points harmonically in the given ratio.

Prove:

I. That any point in the circumference satisfies the given condition.

II. That any point satisfying the given condition lies in the circumference.

I. *Solution by A. E. Jeffrey, Goshen High School, Goshen, Ind.*

Given line AB divided harmonically at C and C′ in the ratio $(m : n)$ and upon CC′ as a diameter a circle is drawn.

I. Let P be any point on the circle (except the extremities of the diameter).

Join P with B and C. Construct an angle BPA′ so that CP is its bisector.

Then PC′ is the bisector of the exterior angle at P (since CPC′ = 90°).

Then A′B is divided harmonically at C and C′.

Hence the proportion	CB : CA′ : : C′B : C′A′.		(1)
	CB : CA : : C′B : C′A.	(Hyp.)	(2)
Divide (1) by (2),	CA : CA′ : : C′A : C′A′.		(3)

Now take (3) by division (CA—CA′) : CA : : (C′A—C′A′) : C′A,
or AA′ : CA : : AA′ : C′A, which can be true only when AA′ = 0.

Therefore PA and PA′ coincide and CP is the bisector of angle APB.

Therefore PA : PB = CA : CB = $m : n$.

II.

To prove any point Q which gives QA : QB = $m : n$ lies on the circle.

Proof: If QA : QB = $m : n$ = CA : CB then the lines QC and QC′ must bisect the int. and ext. angles at Q.

Then the angle CQC′ is a rt. angle (bisectors of sup-adj. angles).

Therefore Q lies on the circle whose diameter is CC′.

Therefore every point on the circle satisfies the equation and every point satisfying the equation is on the circle. Q. E. D.

II. *Solution by R. M. Mathews, Riverside, California.*

II. Construct any triangle ABP such that AP : PB = $m : n$.

Bisect the interior and exterior angles at P, letting the bisectors cut AB internally at X and externally at Y. Since

$$\frac{AX}{XB} = \frac{AY}{YB} = \frac{m}{n},$$

the points X and Y are unique. The $\angle$ XPY is always a right angle;

therefore every P such that PA : PB $= m : n$ is on the circle described with XY as a diameter.

I. Conversely, join any point P on the circle defined above to Y, B and X and draw a line through P to cut YX at A′ so that $\angle$ A′PX = $\angle$XPB. Now, as $\angle$XPY is a right angle (inscribed in a semicircle), PX and PY are the bisectors of the interior and exterior angles at P of triangle A′PB, and so A′ is the fourth harmonic of XY with regard to B, and therefore coincides with A. Accordingly, for every point on the circle PA : PB $= m : n$.

443. *Proposed by Norman Anning, Chilliwack, B. C.*

Triangle ABC is right angled at A. AD is perpendicular to BC and E is a point in BC such that CE = CA. Show that

$$\frac{1}{ED} = \frac{1}{EB} + \frac{1}{EC}.$$

I. *Solution by Hazel Hanson, student at South Dakota State College, Brookings, S. D.*

With the notation given above

$$\overline{AC}^2 = CD \cdot BC = \overline{EC}^2$$

But CD = CE—DE, and BC = CE+EB.

Therefore

$$EB \cdot EC = EC \cdot ED + ED \cdot EB.$$

Dividing by EB · EC · ED gives the required relation

$$\frac{1}{ED} = \frac{1}{EB} + \frac{1}{EC}.$$

II. *Solution by Elizabeth Sargent, Concord, N. H.*

Δ ACE is isosceles. $\angle$ CEA = $\angle$ CAE. $\angle$ DAE comp. $\angle$ CEA; $\angle$ BAE comp. $\angle$ CAE; $\angle$ BAE = $\angle$ DAE. In Δ BAD, since AE bisects $\angle$ BAD, $\frac{BE}{ED} = \frac{BA}{AD}$. $\Delta BAD \sim \Delta BAC$. $\frac{AB}{AD} = \frac{BC}{AC}$ or $\frac{BC}{CE}$.

$$\therefore \frac{BE}{ED} = \frac{BC}{CE}.$$

$$BE \cdot CE = ED \cdot BC = ED(EC+BE) = ED \cdot EC + ED \cdot BE.$$

Dividing by ED · EB · EC, $\frac{1}{ED} = \frac{1}{EB} + \frac{1}{EC}$.

III. *Solution by Elmer Schuyler, Bay Ridge High School, Brooklyn, N. Y.*

Let sides opposite A, B and C be respectively designated by a, b and c.

$$ED = CA - CD = b - \frac{b^2}{a} = \frac{b(a-b)}{a}.$$

$$\frac{1}{ED} = \frac{a}{b(a-b)} = \frac{1}{b} + \frac{1}{a-b}.$$

$$\therefore \frac{1}{ED} = \frac{1}{EC} + \frac{1}{EB}, \text{ since } EC = b \text{ and } EB = a-b.$$

444. *Proposed by A. Heinz, Cameron, Mo.*

The following construction is given without proof in a Chinese book of constructions:

To construct a regular pentagon on AB as a side.

(1) With A and B as centers and AB as a radius, draw circles intersecting at C and D.

(2) With C as a center and radius CA, draw a circle cutting circles A and B at E and F, respectively, and CD at G.

(3) Join EG and FG, produce to meet circles B and A at H and I, respectively.

(4) With H and I as centers and radius AB, draw arcs cutting at J on CD produced.

(5) ABHJIA is the required pentagon.

Prove this construction correct or incorrect—a geometric proof desired.

I. *Solution by James H. Weaver, West Chester, Pa.*

Draw from B to HG the perpendicular BP and let FI, CD and EH intersect AB in R, O and S, respectively. Then the triangle GSR is right and isosceles, and OR = OG = OS. But since AB is equal to the radius of the circle with center C, if we call $AB = 2a$ then $GO = a(2-\sqrt{3}) = SO$. The triangle SPB is also right and isosceles and $SB, = a(3-\sqrt{3})$, is the hypotenuse. Therefore, $SP = PB = \frac{a}{\sqrt{2}}(3-\sqrt{3})$. Also $SG = a\sqrt{2}(2-\sqrt{3})$.

Now in the right triangle BPH, $BH = 2a$. Therefore, $PH = \sqrt{HB^2-PB^2} = a\sqrt{3\sqrt{3}-2}$. But $PS-SG = GP = a/\sqrt{2}(\sqrt{3}-1)$ and $GP + PH = GH = a/\sqrt{2}(\sqrt{3}-1)+a\sqrt{3\sqrt{3}-2}$.

Draw HI which is a diagonal of the pentagon. IGH is an isosceles right triangle. Therefore, $IH = a[(\sqrt{3}-1)+\sqrt{6\sqrt{3}-4}]$.

But if IH were the diagonal of a regular pentagon its value would be $2a(\sqrt{5}+1)$. Therefore the construction is not exact.

II. *Solution by R. M. Mathews, Riverside, California.*

The construction does not give a true regular pentagon.

Let EGH cut AB at X. Since E, C and F are collinear and GC is perpendicular to EF and EC = GC, $\angle HXB = 45°$.

Let CD and AB intersect at M. Taking AB as a unit,

$$XM = MG = 1-\tfrac{1}{2}\sqrt{3}.$$

$$XB = \tfrac{1}{2}+1-\tfrac{1}{2}\sqrt{3} = \tfrac{1}{2}(3-\sqrt{3}).$$

From H draw HY perpendicular to AB produced. Then XY = HY.

If the pentagon be true, $\angle YBH = 72°$ and $\angle BHY = 18°$, which is half the central angle subtended by the side of a regular decagon. Then

$$BH : BY : HY = 1 : \frac{\sqrt{5}-1}{4} : \frac{\sqrt{10+2\sqrt{5}}}{4}.$$

If XY = HY,

$$\tfrac{1}{2}(3-\sqrt{3})+\frac{\sqrt{5}-1}{4}=\frac{\sqrt{10+2\sqrt{5}}}{4},$$ which is not true.

Note: Trigonometric solution of the triangle XBH gives $\angle XHB = 26° 38'$ so $\angle XBH = 108° 22'$, instead of 108°.

III. *Solution by Elmer Schuyler, Bay Ridge High School, Brooklyn, N. Y.*

If the construction is exact the angles of the triangle EBH are $\angle E = 15°$, $\angle EBH = 138°$, $\angle H = 27°$ and side $EB = r\sqrt{3}$ (where r = AB) and $BH = r$. Then the following equation, by the Law of Sines, is true:

$$\frac{\sin 27°}{\sin 15°} = \frac{r\sqrt{3}}{r} = \sqrt{3} = 1.7320508 \ldots .$$

By logarithms,

$$\frac{\sin 27°}{\sin 15°} = 1.754 \ldots .$$

Hence the construction is in error by about .02.

IV. *Solution by Veda Larson, Stoughton, Wis.*

In the construction suggested the angle EGF is a right angle, being

inscribed in a semicircle. This makes the triangle GLM, determined by AB, GF and GE a right isosceles triangle, because the construction is symmetric with respect to CD. Then if P be the intersection of CD and AB, GP = CG—CP = AB—AB sin 60°.

In any regular pentagon ABCDE draw an altitude BR and a diagonal AC cutting BR in T. Upon AC as a diameter construct a circle cutting BR internally in O. Draw CO and AO, producing them to meet ED in S and L, respectively. ΔOSL is then a right isosceles triangle. OR = TR—TO = AB sin 72°—AB sin 54°.

Since GP and OR are homologous sects of the polygons, if AB is unity they should have equal values, if the first construction is correct. The second is correct, and since GP $\neq$ OR, the first construction has been shown incorrect.

Trigonometry.

445. *Proposed by Nelson L. Roray, Metuchen, New Jersey.*

If the bisectors of angles B and C of a triangle intersect the opposite sides in E and F, respectively, prove that EF makes with BC an angle

$$\tan^{-1}\frac{(b-c)\sin A}{(a+b)\cos C+(a+c)\cos B}.$$

I. *Solution by Norman Anning, Chilliwack, B. C.*

Choose BC as x—axis and the perpendicular through A as y—axis. Then the vertices A, B and C are, respectively, $(0, c\sin B)$, $(-c\cos B, 0)$ and $(b\cos C, 0)$. Now BF : FA $= a : b$ since CF bisects $\angle$ BCA. Therefore, the co-ordinates of F are

$$\left(\frac{a : 0+b\ (-c\cos B)}{a+b},\ \frac{ac\sin B+b\ : 0}{a+b}\right) \equiv (x_2, y_2).$$

Similarly, those of E are $\left(\frac{cb\cos C}{a+c},\ \frac{ac\sin B}{a+c}\right) \equiv (x_1, y_1)$.

The tangent of the angle which EF makes with BC is given by

$$\frac{y_1-y_2}{x_1-x_2}=\frac{\frac{ac\sin B}{a+c}-\frac{ac\sin B}{a+b}}{\frac{cb\cos C}{a+c}+\frac{cb\cos B}{a+b}}=,\ \frac{(b-c)\sin A}{(a+b)\cos C+(a+c)\cos B}$$

Q. E. D.

II. *Solution by the Proposer.*

Let the figure be constructed according to the problem and let EF meet CB at G. Take EH perpendicular to BC.

Then

CE/EA $= a/c$, whence CE $=\frac{ab}{a+c}$ and CH $=\frac{ab}{a+c}\cos C$. BH $= a-\frac{ab}{a+c}\cos C$.

$$\frac{AE}{EC}\cdot\frac{CG}{BG}\cdot\frac{BF}{FA}=1,$$

(since CG/BG = CG′/BG′, where G′ is the intersection with BC of the bisector of $\angle$A).

Then

CG/BG $= b/c$ and therefore BG $=\frac{ca}{b-c}$ Also EH $=\frac{ab}{a+c}\sin C$.

$$\therefore \tan G=\frac{EH}{GH}=\frac{(b-c)\sin C}{a+c-(b-c)\cos C}$$

$$=\frac{(b-c)\sin C}{c\cos B+c+c\cos C}\qquad (a\ =\ b\cos C+c\cos B)$$

$$= \frac{(b-c)\sin A}{a\cos B + a + a\cos C}$$

$$= \frac{(b-c)\sin A}{(a+c)\cos B + (a+b)\cos C} \quad (a = b\cos C + c\cos B).$$

Q. E. D.

CREDIT FOR SOLUTIONS.

441\. Norman Anning, S. F. Atwood, Niel Beardsley, T. M. Blakeslee, Grace Boyd, Edward Fleischer, Aaron Freilich, J. Wilbur Haines, R. M. Mathews, Harriet L. Pope, Nelson L. Roray, Elmer Schuyler, H. H. Seidel, Fenton V. Stearns, J. H. Weaver, Henry S. Williams, C. S. Yeh (17)

442\. Mabel G. Burdick, Edward Fleisher, A. E. Jeffrey, R. M. Mathews, Nelson L. Roray, C. S. Yeh, one incomplete solution. (7)

443 Norman Anning, Edith H. Barker, Grace Boyd, Walker Cisler, Minnie Dingee, Edward Fleischer, Aaron Freilich, J. Wilbur Haines. Hazel Hanson, A. E. Jeffrey, M. Helen Kelley, Veda Larson, R. M. Mathews, R. H. Montgomery, Harriet L. Pope, Nelson L. Roray, Elizabeth Sargent, Elmer Schuyler, H. H. Seidel, L. B. White, George W. Wriston, C. S. Yeh, Mabel G. Burdick. (23)

444\. Veda Larson, R. M. Mathews, Nelson L. Roray, Elmer Schuyler, Jas. H. Weaver, one incorrect solution. (6)

445\. Norman Anning, R. M. Mathews, Nelson L. Roray, Elmer Schuyler, one incorrect solution. (5)
Total number of solutions, 58.

PROBLEMS FOR SOLUTION.

Correction.

452\. Attention is here called to an error in the statement of problem 452 in the December number. The equation should read $e^{\frac{2m}{v}} = \frac{v+m}{v-m}$.

455\. Problem 455 in the December number was also misstated and should read as follows: Given $\tan\alpha = \cos\epsilon\tan\lambda$, and $\epsilon = 23°\,27'$. —Editor.

Will contributors please note the new and correct form of the equation and send solutions accordingly?

Algebra.

456\. *Proposed by H. Perkins, Battleford High School, Battleford, Sask.*

Find the sum of n terms of the sequence of fractions

$$\frac{1}{1+x},\ \frac{2x}{1+x^2},\ \frac{4x^3}{1+x^4},\ \frac{8x^7}{1+x^8},\ \ldots\ldots$$

457\. *Proposed by Nelson L. Roray, Metuchen, New Jersey.*

Jones settled an annuity upon his three daughters to be divided in the same ratio as their ages. At the first payment the oldest was entitled to one-half the annuity. When the sixth payment was made Martha received one dollar less than she had the first year, Phoebe 1/7 less than the first year and Mary twice as much. What was the amount of the annuity?—From a recent examination paper.

Geometry.

458\. *Proposed by Marie Brown, St. Louis, Mo.*

Place a six inch sphere in the corner of a room, tangent to the three sides. What is the size of the sphere that may be placed behind it tangent to it and the three sides?

459. *Proposed by Norman Anning, Chilliwack, B. C.*

The sides, in order, of a cyclic pentagon are 4, 6, 4, 6, 4. Show that the diagonals are integral.

460. *Proposed by A. E. Jeffrey, Goshen (Ind.) High School.*

AB is the diameter of a semicircle and at any point C on AB the perpendicular CD is erected to AB meeting the arc at D. Also on AC and BC as diameters circles are drawn.

The two circles which can be drawn, each tangent to two of the semicircles and tangent to CD are equal.

SCIENCE QUESTIONS.

By Franklin T. Jones,
University School, Cleveland, Ohio.

Readers of School Science and Mathematics *are invited to propose questions for solution—scientific or pedagogical—and to answer questions proposed by others or by themselves. Kindly address all communications to Franklin T. Jones, University School, Cleveland, Ohio.*

Questions and Problems for Solution.

204. *Proposed by E. E. Wolfe, Pittsburgh, Pa.*

A block floats with one-half submerged in water. How much of its volume will be under water when a liquid (immiscible with water) of specific gravity .25 is poured on so as to cover the entire block?

205. *Proposed by W. L. Baughman, East St. Louis, Ill.*

A light wire is stretched over two smooth pulleys at a distance of 10 feet from each other in the same horizontal line, and has 112 pounds hung at each end. What weight hung at the middle of the wire will cause it to sag one inch?

The following list of questions was given in examination for the second semester's work by A. C. Norris in Rockford High School, May, 1915. It will be of interest to physics teachers.

1. Define and give illustrations of (*a*) conduction, (*b*) convection and (*c*) radiation.
2. A given mass of gas has a volume of 10,000 cubic feet at 0° C. Find its volume at +10° C. and at —10° C. State the law governing these problems.
3. State two ways by which the boiling point of water may be raised.
4. Reduce —17° C. to F°; +21° F. to C°.
5. State and illustrate the laws of evaporation.
6. Define light, color, shadow, spectrum, and lens.
7. (*a*) Describe how the velocity of light was first discovered. (*b*) The nearest fixed star is four light years away. How many miles?
8. The page of a book is held one foot from a given light. It is then removed to a distance of five feet. Compare the intensity of light on the page in the two cases.
9. State the law of magnetic attraction and repulsion.
10. (*a*) Make drawings to illustrate magnetic fields between like poles; also unlike poles. (*b*) Explain magnetic dip. (*c*) Explain angle of declination.

Willis E. Tower of Englewood High School, Chicago, Ill., in preparing a paper on "The Effective Teaching of Physics in High Schools," sent out the following list of questions in order to be able to present plans and methods now in successful use.

1. What is the most effective plan that you have used in securing effective results in recitation?

2. What general plan do you find gives the best results in laboratory work?
3. What tests do you give How many? When?
4. Do you require exercises handed in at each recitation?
5. What proportion of failures may reasonably be expected in physics classes?
6. What special plan or device have you found of especial help in physics teaching?

Please answer questions numbered 206 and 207 in the following list.

HARVARD—CHEMISTRY—NEW PLAN.

(One Hour and a Half.)

All notebooks must be handed in at the laboratory examination and must be claimed when it is over. Candidates should answer the first five questions, and should then select five from the remaining questions.

1. State how sulphur dioxide may be prepared free from other gases, and describe its most important properties and reactions.
2. Write the following equations in complete form, using formulas:
   (*a*) Sodium sulphide + hydrochloric acid = ?
   (*b*) Hydrogen gas + oxygen gas = ?
   (*c*) Silver sulphate + ammonium chloride = ?

206. How many cubic centimeters of hydrochloric acid gas measured under standard conditions can be obtained from 7.00 grams of aluminum chloride, $AlCl_3 \cdot 6H_2O$?

$H = 1 \quad O = 16 \quad Al = 24 \quad Cl = 35.5$

One liter of hydrochloric acid gas at 0° C. and 760 mm. weighs 1.64 grams.

4. Define *acid, base, salt,* and write in each case an equation illustrating how one such substance may be made.
5. Give an example of a chemical change produced by (*a*) heat, (*b*) light, (*c*) electrical energy.

207. Three grams of sodium chloride yield 7.35 grams of silver chloride. Calculate from these data the atomic weight of sodium.

$Cl = 35.5 \quad Ag = 107.9$

7. (*a*) Give names of two compounds, not mixtures, that may be used as high explosives. (*b*) How does their action differ chemically from that of gunpowder?
8. If excess of hydrochloric acid solution is added to sodium sulphate, and the mixture evaporated to dryness, will pure sodium chloride be left behind? If excess of sulphuric acid is added to sodium chloride, and the mixture evaporated to dryness, will pure sodium sulphate be left behind? Give a reason for each answer.

Solutions and Answers.

191. *Harvard Physical Science, June, 1887.*

If a carriage wheel be resting upright upon the ground, and be prevented from slipping at the bottom, how great a force applied horizontally at the top will just neutralize a force equal to the weight of 50 pounds applied horizontally in the opposite direction at the center of the wheel?

Solution by Robt. W. Boreman, Parkersburg, W. Va.

Let a = radius of wheel and x the horizontal force.

$50a = x \cdot 2a \quad x = 25$ pounds or 804 poundals.

192. Into 110 grams of water at 15° C. contained in a vessel the thermal capacity of which is equal to that of 10 grams of water, are put 200 grams of a certain solid at 100° C., and the resulting temperature of the whole is 25° C. Calculate the specific heat of the solid.

Solution by R. W. Boreman.

$110+10 = 120$g. $\quad 25°-15° = 10°$.

$120 \times 10 = 1{,}200$ calories absorbed by water and vessel.

Let x = required specific heat of unknown solid.

$200(100°-25°)x$ = total heat given off by solid = 1,200.

$x = .08$, specific heat of solid.

193. *Proposed from Bloomington, Ill., in an unsigned letter.*

I would like to have some physics teacher answer this question asked me by a boy in my physics class:

Induced E. M. F. is proportional to the rate of change in the magnetic flux. In A. C. transformers this change is caused by the alternations of the current and so the rate of change seems to be determined by the frequency of the alternations, yet the ratio of the secondary E. M. F. to the primary is given as the ratio of the number of turns in the secondary to the number in the primary, regardless of the number of cycles per second.

Answer by Roy E. Jensen, Belle Vernon, Pa.

Also answered by R. W. Boreman and Wm. B. Borgers.

Induced E. M. F. is proportional to the number of lines of magnetic force cut by a conductor per second. In the A. C. transformer, the cutting of the lines of force is effected not by motion of conductor, but by causing the lines of force to be created and made to thread the stationary conductor. Therefore the strength of the magnetic field formed by the primary coil will determine the E. M. F. in the secondary coil.

The strength of a magnetic field is not influenced by element of time but depends only on the number of ampere turns that surround the core. Therefore the induced E. M. F. in A. C. transformers will depend only upon the number of turns in the two coils.

Wm. B. Borgers, Chicago, Ill., says in part:

The fact that E is proportional to the number of turns of wire does not prevent its being at the same time proportional to some other variable. You could illustrate this for the boy by reminding him how the intensity of illumination is proportional to the amount of light emitted by the source, but at the same time, is inversely proportional to the square of the distance; or how the force exerted by the steam on a piston is proportional both to the pressure and the area, and hence to their product.

194. *From an entrance examination of Columbia University.*

A man weighing 150 pounds carries a load of 70 pounds up an incline of one in ten. If he walks at the rate of one thousand feet per minute, what horse power is he exerting? Answer to be in foot pounds per second.

[Is the answer reasonable? Should problems with unreasonable answers be given, especially on examinations?—EDITOR.]

Solution by R. W. Boreman.

$70+150=220$ pounds.

$1{,}000\div10=100$ feet, vertical distance he would raise the weight in one minute.

$220\times100=22{,}000$ foot pounds exerted per minute.

$$\text{h.p.}=\frac{\text{No. of foot pounds}}{550\times\text{time in seconds}}=\frac{22{,}000}{550\times60}=\tfrac{2}{3}\text{ h. p. or } .666\text{ h. p.}$$

The answer is reasonable to the extent that a man might exert two-thirds of a h.p. for a short space of time, but the statement that he *walked* with a heavy burden at the rate of 1,000 feet per minute is absurd.

Comment by Roy E. Jensen.

I consider problem 194 very poorly stated as well as unreasonable and should be eliminated from any examination list.

195. *Suggested by an Associated Press Dispatch, dated St. Louis, Mo., May 2, 1915.*

At Tipton, hailstones were found which measured 8½ inches in diameter and weighed half a pound. Hailstones fell as large as baseballs.

[How much would such hailstones weigh?—EDITOR.]

Solution by Roy E. Jensen.

Also solved by R. W. Boreman.

8½ inches $=21.59$ cm. $=$ diameter of sphere.

Vol. of sphere $= D^3 \times \frac{\pi}{6} = 21.59^3 \times .5236 = 5{,}269.13$ c. cm.

Density of ice $= .917$ $\therefore$ Wt. $= 5{,}269.13 \times .917$ grams $= 4{,}831.79$ grams $= 4.83179$ kg.

4.83179×2.2 lbs. $= 10.63$ lbs.

Note.—Hailstones not being the most compact form of ice would probably have a slightly lower density than .917.

Note by Editor.—The journalist who saw these hailstones was evidently not hit by one of them.

196. *From a Yale University Physics entrance Examination:*

A pneumatic tire is blown up to a pressure of 75 pounds per square inch. In the pump employed, the piston area is 3 square inches, and its travel is 15 inches. At what point in the piston travel will the tire valve open on the last stroke? What force must be exerted on the piston at that moment? Explain the heating of the rubber tube connecting the pump and the tire in this process.

Solution by Wm. B. Borgers.

Also solved by Roy E. Jensen and R. W. Boreman.

Let us assume: That when the piston reaches the end of its travel, no air is left beyond it in the cylinder; that the pressure needed to compress the valve spring and overcome friction and adhesion is q; that "a pressure of 75 pounds" means gauge pressure, and that atmospheric is 15; that Boyle's law is strictly true, and that no material temperature difference between tire and cylinder occurs; that the volume of the tire is v cubic inches, and that it does not stretch during the last stroke.

Let x be the volume of air in the cylinder when the valve opens, and d the inches of stroke remaining; then $x = 3d$. Let p be the pressure in the tire when the valve opens. Then $p : 90 = v : v+x$, or $p = \dfrac{90v}{v+x}$

At the same time the cylinder pressure will be $p+q$.

$x : 45 = 15 : \dfrac{90v}{v+x} + q$, where 45 cubic inches is the whole volume of the cylinder. Then $\dfrac{90vx}{v+x} + qx = 675$; $qx^2 + (90v+qv-675)x - 675v = 0.$ (1).

Solving and dividing by 3, since $d = \frac{1}{3}x$,

$$d = \frac{-(90v+qv-675) \pm \sqrt{(90v+qv-675)^2 + 2{,}700qv}}{6q}$$

If q were negligibly small (which was, I suspect, intended, though it is by no means so), (1) would become $(90v-675)x = 675$, $x = \dfrac{15v}{2v-15}$ (2).

If at the same time v were indefinitely large, so that the tire pressure would not materially increase during the last stroke, then (2) would become $x = \dfrac{15v}{2v} = 7.5$, or $d = 2.5$ inches, which is probably the answer the examiner expected.

10 pounds per square inch would be a reasonable value for q, and 1,000 cubic inches for v. In that case, $d = \dfrac{-99{,}325 \pm \sqrt{99{,}325^2 + 27{,}000{,}000}}{60}$ $= 2.264$ inches.

With the same value of q, if v were indefinitely large, $d =$

$$\frac{-(100v-675) \pm \sqrt{10{,}000v^2 - 108{,}000v + 455{,}625}}{60}$$

The expression under the radical equals $(100v-540)^2 + 164{,}025$. So that as v indefinitely increases, the radical term approaches $100v-540$, and therefore the numerator approaches 135. This makes d approach 2.25 inches.

The pressure exerted by the hand (or the engine, more likely) is

$p+q$ minus the atmospheric pressure, or $\frac{90v}{v+x}+q-15$, which can be evaluated as soon as values of v and q are chosen. And the force is pressure $\times$ area, or $3(p+q-15)$. With v indefinitely large, and q indefinitely small, this becomes 225 pounds, probably the answer expected by the examiner. With $q=10$ pounds per square inch, $f=255$ pounds. With $q=10$ and $v=1{,}000$, $f=253.1$ pounds.

The last part may be answered thus: When an air molecule rebounds from a stationary wall, its velocity is unchanged except in direction; but when it rebounds from the approaching piston, the piston velocity is added to that of the molecule. The molecules then strike the walls of the tube with this increased velocity, and hence set the molecules of rubber into more rapid motion—i. e., raise the temperature of the rubber.

A VIBRATION ELECTROMETER.

Any alternating current measurement which makes use of a null method requires an instrument which will detect small alternating currents or voltages. One of the first instruments used for this purpose was the telephone. This is very sensitive between the frequencies of 500 and 3,000 cycles per second, but at frequencies below 500 cycles the sensitiveness decreases rapidly with the frequency, so that it is very insensitive at frequencies below 100 cycles. It also responds to the harmonics of the current as readily as to the fundamental.

As a null instrument, a vibration galvanometer is often much more satisfactory than a telephone. The moving system of a vibrating galvanometer is adjusted to have the same period as that of the current to be detected, so that any harmonics in the current produce very little effect upon the deflection of the instrument. Also most vibration galvanometers have their maximum sensitiveness at low frequencies (50 to 200 cycles) though at least one form may be had which will go to frequencies as high as 3,000 cycles. Since the impedance of these instruments is relatively low, they require an appreciable current to produce a deflection which can be observed. Hence in bridges where the impedance of the arms is very high, they are not very sensitive.

The vibration electrometer described in Scientific Paper No. 239, recently issued by the Bureau of Standards, of the Department of Commerce, was designed as a vibrating instrument having an impedance much higher than a telephone or vibration galvanometer. The need arose in connection with the measurement of some very low capacities at low frequencies. Its usefulness is limited to those cases where it is desired to detect very small currents at low frequencies. Its principal use is as a detecting instrument in a bridge having very high impedances in the arms.

The instrument is a modification of a quadrant electrometer. Instead of the quadrants there are four vertical plates, while a thin vertical vane of twice the area of a single plate corresponds to the needle of the electrometer. Two plates, separated by a narrow vertical slit, are in one plane, while opposite them in a parallel plane are the other plates. Midway between the planes is the aluminum vane, which is suspended by a bifilar suspension. This vane is maintained at constant potential by a battery, while an alternating voltage having the same period as the natural period of the vibrating system is applied to the plates. This causes mechanical forces to be applied to the vane, due to electrostatic attractions and repulsions which will set the vane in vibration. Since these forces are small, it is necessary that the damping shall be small.

In addition to so designing the suspension that there is very little loss of energy in it, it is necessary to keep the instrument in a vacuum.

The form of the instrument is such that the capacities can be approximately computed. Hence, it is possible to develop the mathematical theory of its behavior. This has been done and the conclusions reached have been checked by experiment. The important conclusions are as follows:

1. The frequency at which maximum deflection is obtained depends upon the potential of the vane. As the potential of the vane is increased, the frequency at which maximum deflection is obtained is decreased.

2. The deflection for a given voltage is inversely proportional to the damping.

3. As the damping is decreased, the tuning becomes sharper.

4. The power required to give unit deflection when the applied emf is in resonance with the instrument decreases in the same ratio as the damping.

Experimentally it has been found that the instrument will detect a current as low as 10-11 ampere.

NOTE ON TRIANGLES WHOSE SIDES ARE WHOLE NUMBERS.

BY NORMAN ANNING,
Clayburn, B. C.

90°-TRIANGLE.

$$1+i^2=0.$$
$$(x+iy)\,(x-iy)=x^2+y^2.$$
$$(x+iy)^2\,(x-iy)^2=(x^2+y^2)^2.$$
$$[(x^2-y^2)+i(2xy)][(x^2-y^2)-i(2xy)]=(x^2+y^2)^2.$$
$$(x^2-y^2)^2+(2xy)^2=(x^2+y^2)^2.$$

The numbers, x^2-y^2, $2xy$, and x^2+y^2 are sides of a 90°-triangle. Distinct solutions are given when x and y are relatively prime and not congruent, modulo 2. Every prime of form $4k+1$ will be the longest side of one such triangle. In a complete list of distinct solutions any composite number, all of whose factors are of the form $4k+1$, will appear more than once as longest side, provided that its factors are not all alike.

Examples:

$$3^2+4^2=5^2.$$
$$5^2+12^2=13^2.$$
$$15^2+8^2=17^2.$$
$$16^2+63^2=65^2.$$
$$33^2+56^2=65^2.$$
$$119^2+120^2=169^2.$$

$\frac{119+120}{169}$ is a convergent to $\sqrt{2}$.

120°-TRIANGLE.

$$1+w+w^2=0.$$
$$(x-wy)\,(x-w^2y)=(x^2+xy+y^2).$$
$$(x-wy)^2\,(x-w^2y)^2=(x^2+xy+y^2)^2.$$
$$[(x^2-y^2)-w(2xy+y^2)][(x^2-y^2)-w^2(2xy+y^2)]=(x^2+xy+y^2)^2.$$
$$(x^2-y^2)^2+(x^2-y^2)\,(2xy+y^2)+(2xy+y^2)^2=(x^2+xy+y^2)^2.$$

The numbers, x^2-y^2, $2xy+y^2$, and x^2+xy+y^2 are sides of a 120°-triangle Distinct solutions are given when x and y are relatively prime and not congruent, modulo 3. Every prime of form $6k+1$ will be the longest side of one such triangle. In a complete list of distinct solutions any composite number, all of whose factors are of the form $6k+1$, will appear more than once as longest side, provided that its factors are not all alike.

Examples:

$$
\begin{aligned}
3^2 + 3\cdot 5 + 5^2 &= 7^2.\\
7^2 + 7\cdot 8 + 8^2 &= 13^2.\\
5^2 + 5\cdot 16 + 16^2 &= 19^2.\\
11^2 + 11\cdot 85 + 85^2 &= 91^2.\\
19^2 + 19\cdot 80 + 80^2 &= 91^2.\\
104^2 + 104\cdot 105 + 105^2 &= 181^2.
\end{aligned}
$$

$\frac{181+181}{104+105}$ is a convergent to $\sqrt{3}$.

Each solution for the 120°-triangle yields two solutions for the 60°-triangle, viz:

$$(x^2-y^2),\ (2xy+x^2),\ (x^2+xy+y^2)\ \textit{and}$$
$$(2xy+x^2),\ (2xy+y^2),\ (x^2+xy+y^2).$$

Ex. from (3, 5, 7) comes (3, 8, 7) and (5, 8, 7).

KANSAS ASSOCIATION OF MATHEMATICS TEACHERS.

The fourteenth regular meeting of the Association was held Friday, November 12, 1915, in conjunction with the Kansas State Teachers' Association. The meeting of the mathematics teachers was preceded by a luncheon at which time about sixty teachers renewed old friendships and formed new ones. It was decided that a luncheon should form part of the program for the 1916 meeting. After the luncheon the meeting opened with a business session. Prof. U. G. Mitchell made the report for the committee appointed last year to examine geometry texts submitted for state adoption. The committee gave Wentworth-Smith *Plane and Solid Geometry* as its first choice and Ford and Ammerman's *Plane and Solid Geometry* as second choice. The officers for the ensuing year are:

President—Mrs. Mary W. Newson, Topeka.
Vice-President—Emma Hyde, Kansas City, Kan.
Secretary-Treasurer—Eleanora Harris, Hutchinson.

The first number on the program was a report of the recent meeting of the Missouri Society of Teachers of Mathematics and Science. This report was given by Miss Emma Hyde. Miss Elizabeth G. Flagg read a paper, "A Different Point of View." Her paper was based on work taken by her in California the past summer. The last half of the program was given over to the reports of the Committees on Junior and Senior High School Courses. The reports were discussed by the members of the Association but no definite action was taken other than that the chair was asked to name a committee to consider the subject further.

CLOSE ESTIMATING.

A geologist of the United States Geological Survey once estimated 3,000 feet as the necessary depth to drill in a certain locality to find water, with the result of less than one per cent of error, a flow measuring half a million gallons a day having been struck at a depth of 2,987 feet. In another branch of the work of the Survey, that of estimating at the close of the calendar year the production of the various minerals during that year, even this percentage of error is being reduced. The Survey's estimate on January 1, 1915, of the production of iron ore was 41,440,000 long tons; the actual figures received from all the companies are now seen to be 41,439,761 long tons, a difference of only 239 tons.

ARTICLES IN CURRENT PERIODICALS.

American Forestry for November; *Washington, D. C.; $3.00 per year, 25 cents a copy:* "The Sugar Maple—Identification and Characteristics" (five illustrations); "Commercial Uses of Sugar Maple" (fourteen illustrations), Hu Maxwell; "Maple Sugar Making" (two illustrations); "The Reforestation Movement in China" (eleven illustrations), W. F. Sherfesee; "Suggestions for Using Cypress Knees" (eleven illustrations), Howard F. Weiss; "Growing Pine at a Profit" (four illustrations), J. R. Simmons; "Logging Rasak and Lagan" (five illustrations), T. R. Helms; "A Trip on the Apache National Forest" (six illustrations), A. P. W.; "Ornamental and Shade Trees—Common-Sense Labels on Park Trees," J. J. Levison.

American Mathematical Monthly for November; *5548 Kenwood Ave., Chicago; $2.00 per year:* "The Teaching of Mathematics," H. E. Slaught; "History of Zeno's Arguments (Concluded)," Florian Cajori; "Mathematical Meetings in California," E. R. Hedrick; "History of Mathematics," G. A. Miller; "On the Circles of Appolonius (Concluded)," Nathan Altshiller.

American Naturalist for December; *Garrison, N. Y.; $4.00 per year, 40 cents a copy:* "Some Experiments in Mass Selection," W. E. Castle; "The Inheritance of Black-eyed White Spotting in Mice," Dr. C. C. Little; "The F_1 Blend Accompanied by Genic Purity," Dr. H. H. Laughlin; "The Population of the 'Blanket Algae' of Freshwater Pools," Emilie Louise.

Journal of Geography for December; *Madison, Wis.; $1.00 per year, 15 cents a copy:* "The Cultural and the Training Value of Geography," Bertha Henderson; "Where Our Principal Minerals Come From," F. E. Williams; "The Carnegie Institutions's Atlas of the Historical Geography of the United States," Charles O. Paullin; "Types of Laboratory Exercises for High School Geography," Cassa C. Benham; "Texts for Home Geography," Philip Emerson.

Nature-Study Review for November; *Ithaca, N. Y.; $1.00 per year, 15 cents a copy:* "The Organization of Nature Study," John Dearness; "Bulb Planting in the Kindergarten," Lucile P. Hosse; "School Gardening in the Philippines," N. H. Foreman; "The School Garden, a Laboratory for Industrial Education," Alice V. Joyce; "The Black Duck," "The Lapland Longspur," Walter K. Putney; "A Sixth Sense in Birds and Mammals," G. O. Shields; "Nature-Study on an Old Time Farm," John MacDougall; "Nature Play in Los Angeles Schools," Frances Conrad.

Photo-Era for December; *383 Boylston St., Boston; $1.50 per year, 15 cents a copy:* "The Education of the Photographic Artist," Paul Lewis Anderson; "Some Notes on Winter-Subjects," William S. Davis; "Lens Facts and Fallacies," J. A. Dawes; "Artistic Feeling in the Snapshot," Will W. Todd; "How I Reproduce Broken Ambrotypes," L. C. Bishop; "Panchromatic Photography by Gaslight," W. R. Bradford.

Physical Review for November; *Ithaca, N. Y.; $6.00 per year, 60 cents a copy:* "Physical Photometry with a Thermopile Artificial Eye," Herbert E. Ives and E. F. Kingsbury; "A Precision Artificial Eye," Herbert E. Ives; "The Mobility of the Positive and Negative Ion at Different Temperatures and at Constant Gas Density," Henry A. Erikson; "A Lens Refractometer," H. F. Dawes; "A New Fluorescence Spectrum of Uranyl Ammonio-Chloride," Edward L. Nichols and Ernest Merritt; "The Variation of the Specific Heat of Solids with Temperature," Arthur H. Compton; "A Highly Sensitive Electrometer," A. L. Parson; "The Influence of the Metallic Ion in Electrolytic Solutions upon the Potential Differences between the Solutions and a Metallic Electrode," Florella K. Finney; "Saturation Value of the Intensity of Magnetization and the Theory of the Hysteresis Loop," E. H. Williams.

Popular Astronomy for December; *Northfield, Minnesota; $3.50 per year, 35 cents a copy:* "Eighteenth Meeting of the American Astronomical

Society"; "Sirius," Charles Nevers Holmes; "Report on Mars, No. 12," William H. Pickering; "The Position of the Sun in Space," William Albert Mason; "Light Curves of the Moon and the Zodiacal Light Compared," W. E. Glanville.

Scientific Monthly for December; *Garrison, N. Y.; $3.00 per year; 30 cents a copy:* "The Inside of a Great Medical Discovery," Dr. Aristides Agramonte; "The Evolution of the Stars and the Formation of the Earth," Dr. William Wallace Campbell; "A Metrical Tragedy," Dr. Jos. V. Collins; "Why Certain Plants are Acrid," William R. Lazenby; "How Our Ancestors Were Cured," Professor Carl Holliday; "A Visit to Oeningen," Professor T. D. A. Cockerell; "The Theory and Practise of Frost Fighting," Professor Alexander McAdie.

Zeitschrift für Mathematischen und Naturwusenschaft lichen Unterricht Aller Schulgattungen for September; *B. G. Teubner, Leipsic, Germany; 12 numbers, M. 12 per year:* "Die Stellung der Mathematik im Unterrichtsbetriebe der höheren Schulen," Prof. Otto Hesse; "Zur Geschichte, Theorie, und Praxis der Camera obscura," Privat dozent Dr. J. Würschmidt; "Zu meinem Aufsatz: Konvexe pseudoreguläre Polyeder," O. Ransenberger, for October; "Die Ausbildung der Mathematiklehrer an den höheren Schulen Deutschlands," Oberrealschuldirektor Dr. W. Lietzmann; "Zahlwörter und Positionssystem," Dr. A. Schülke; "Graphische Darstellung zur harmonischen Teilung," Oberlehrer Dr. F. A. Jungbluth.

Q.: Why are porous water vessels cooler than the air around?

A.: Because porous is a substance which gives out heat and so a vessel of porous would be colder than the air.

BOOKS RECEIVED.

The Elements of Surveying and Deodesy, by Wm. C. Popplewell, University of Manchester. xi+244 pages. 14.5x22.5 cm. Cloth. 1915. $2.25 net. Longmans, Green & Co., New York City.

Bacteriological Methods in Food and Drugs Laboratories, with an Introduction to Micro-Analytical Methods, by Albert Schneider, Columbia University. viii+288 pages. 14x20 cm. Cloth. 1915. $2.50. P. Blakiston's Son & Co., Philadelphia.

Treatise on Light, by R. A. Houston, University of Glasgow. Pages xi+478. 14.5x22.5 cm. Cloth. 1915. $2.25 net. Longmans, Green & Co., New York City.

How to Study and What to Study, by Richard L. Sandwick, Deerfield-Shields High School, Highland Park, Ill. Pages v+170. 12x17 cm. Cloth. 1915. 60 cents. D. C. Heath & Co.

The Orders of Architecture, by A. Benton Greenburg, Stuyvesant High School, New York City. 20 full page loose problem sheets. 20x26.5 cm. Paper. 1915. 50 cents. John Wiley and Sons, New York City.

Elementary Lessons in Electricity and Magnetism, by Sylvanus P. Thompson, University of London. 7th Edition. Pages xv+706. 14x20 cm. Cloth. 1915. The Macmillan Co., New York City.

Mathematical Monographs, No. 16: Diophantine Analysis, by Robert D. Carmichael, University of Illinois. Pages vi+118. 15x23.5 cm. Cloth. 191. $1.25. John Wiley and Sons, New York City.

Teaching, Its Aims and Methods, by Levi Seeley, New Jersey State Normal School. Pages xi+320. 13x19 cm. Cloth. 1915. $1.25. Hinds, Noble, and Eldridge, New York City.

High School and Class Management, by Horace A. Hollister, University of Illinois. xvii+314. Cloth. 1915. $1.25. D. C. Heath & Co., Boston.

The Wheat Industry, for Use in Schools, by N. A. Bengtson and Donee Griffith, University of Nebraska. Pages xiii+341. 13.5x19 cm. Cloth. 1915. The Macmillan Co., New York City.

Analytical Geometry, by H. B. Phillips, Massachusetts Institute of Technology. Pages vii+197. 13x19 cm. Cloth. 1915. $1.50 net. John Wiley and Sons, New York City.

The Plant Note Book, by Anna Botsford Comstock, Cornell University. 126 pages. 12x19 cm. Paper. 1915. 30 cents. Comstock Publishing Co., Ithaca, N. Y.

Laboratory Accommodation in Constitution and High Schools and Collegiate Institutes, by George A. Cornish, University of Toronto. Pages viii+144. 16.5x25 cm. Cloth. 1915. L. K. Cameron, Toronto.

Plain and Spherical Trigonometry and Tables, by George Wentworth and David Eugene Smith. Pages v+230+26+104. 15.5x24 cm. Cloth. 1915. $1.10. Ginn & Co., Boston.

Q.: How is a sound wave measured?
A.: From compensation to compensation.

Q.: How is simple harmonic motion shown?
A.: By means of an Elyciptale Shadow.

TERRESTRIAL MAGNETISM.

The United States Coast and Geodetic Survey, Department of Commerce, has issued as Serial No. 3, Special Publication No. 25, a quarto pamphlet of 69 pages entitled *Results of Magnetic Observations made by the United States Coast and Geodetic Survey in 1914*, by D. L. Hazard.

This publication contains the results of magnetic observations made on land and at sea during the calendar year of 1914, together with descriptions of the stations occupied. Results are given for 385 stations in 289 localities, including an investigation of areas of marked local disturbance in Iowa and Minnesota. There is presented in tabular form a comparison of the declination results at 76 repeat stations with the results of earlier observations in the same localities. The results have been corrected to reduce them to the provisional international standard of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington.

The stations described are located in 33 states and territories, including Arizona, Alabama, Alaska, Arkansas, California, Colorado, Delaware, Florida, Georgia, Idaho, Illinois, Iowa, Louisiana, Maine, Massachusetts, Minnesota Mississippi, Montana, Nebraska, New Hampshire, New Jersey, New Mexico, New York, North Dakota, Oklahoma, Oregon, Pennsylvania, South Dakota, Tennessee, Texas, Vermont, Washington and Wisconsin.

Besides the scientific value of these observations, this work is of practical utility to engineers and surveyors, and particularly to those interested in retracing old property lines. In the early days and even more recently these lines were run with the compass almost exclusively and to return them a knowledge of the variation of the compass at the date of survey is essential.

The volume will be supplied without charge to persons interested by application to the Division of Publications, Department of Commerce.

BOOK REVIEWS.

Elementary Algebra, First Year Course, by Florian Cajori, Colorado College, and Letitia R. Odell, North Side High School, Denver, Colo. Pages vii+206. 13×19 cm. 65 cents. 1915. The Macmillan Company, New York.

It is the purpose of this book to present algebra to beginners in a simpler, clearer, and more practical form than is usually found in school texts. Certain redundant terms and notations are omitted. An intimate connection between algebra and arithmetic is maintained, and the mechanical manipulation of algebraic expressions is reduced to a small amount. Stress is laid on practical applications to problems arising in business, and graphs are given a practical application. There are several full page pictures of mathematicians and a number of historical notes. H. E. C.

Plane Geometry, by John W. Young, Professor of Mathematics in Dartmouth College, and Albert J. Schwartz, Instructor in Mathematics in the William McKinley High School, St. Louis, Mo. Pages x+223. 13×19 cm. $1.00. 1915. Henry Holt & Company, New York.

There are a number of interesting features in this book. From the beginning considerable use is made of translation and rotation. Symmetry is defined early and used systematically in the proofs. Some of the properties of circles are developed early and used frequently. Many familiar theorems are omitted, probably with advantage to all concerned. Construction and auxiliary lines are printed in green. The incommensurable cases are well treated on purely practical grounds. There is a large number of exercises in construction and drawing. There is little connection with algebra and a limited use of angle functions. It seems as if the authors have succeeded in subordinating formalism, and that pupils should rather easily gain a thorough understanding of the fundamental notions of geometry, learn methods of attack, and become proficient in the formal proof of theorems. H. E. C.

First Year Mathematics, by Ernst R. Breslich, Head of the Department of Mathematics in the University High School, The University of Chicago. Pages xxiv+344. 14×20 cm. 1915. $1.00 and postage. The University of Chicago Press.

The question of algebra first or geometry first was under consideration for some years. In general, algebra, abstract and enigmatical, is now the first year allotment of high school pupils. Fortunately, a new question is under consideration now, and the scholarly work of Mr. Breslich and his colleagues makes it evident that algebra and geometry can be united in a single course of instruction. The subjects are so skillfully interwoven that the reader has no jarring sense of going from one branch of mathematics to another.

Thirteen years of trying out material and revising the text has produced a book which can be used by any teacher of secondary mathematics with little difficulty. It surely will not seem like a new and strange body of mathematics to the teacher, and the clear statements and explanations can be readily understood by the pupil. The main topics of the usual course in first year algebra are well covered, and not till the end of the year do the geometrical demonstrations take the form of logical proofs. Good problem material, exercises requiring measurement and drawing, nearly three hundred well-drawn figures, graphs an integral part of the work, thirteen portraits of mathematicians as inserts with an account

of their life and work, are some of the features of this book which marks a real advance in mathematics teaching. H. E. C.

Mathematical Tables, by Mansfield Merriman, Editor-in-Chief of the American Civil Engineers' Pocket Book. Pages vi+67. 13×19 cm. 1915. John Wiley & Sons, Inc., New York.

These tables have a form which is approved in the work of practical computation, and are very convenient for the use of students in the solution of problems. The explanations of the tables make them easily understood, and the remarks concerning the degree of precision are of practical value. Included in the thirty-five tables are reciprocals, squares, square roots, cubes, cube roots, three-halves powers, fifth powers, areas and circumferences of circles, volumes of spheres, logarithms, trigonometrical functions, lengths, areas, speed and velocity, weight, energy or work, pressure, and power. H. E. C.

Analytic Mechanics, by John A. Miller, Professor of Mathematics, and Scott B. Lilly, Assistant Professor of Engineering, Swathmore College. Pages xv+297. 14×21 cm. 1915. D. C. Heath & Co., New York.

The subject-matter of this introduction to the study of mechanics includes the fundamental principles of mechanics used by students in theoretical physics, in the various branches of engineering, and in celestial mechanics. While no attempt is made to develop special methods used in some branches of these sciences, the fields in which certain processes are employed are pointed out and references to more detailed information are given. The illustrative problems and the exercises are chosen largely from engineering fields, and are real problems dealing with real structures and

real machines. There are many directions and cautions to the student regarding the application of the fundamental theorems to practical problems, which are of real value since they are fundamental discussions of the strength and limitations of basic methods. H. E. C.

A Practical Algebra for Beginners, by Thirmuthis A. Brookman, formerly Head of the Department of Mathematics, Upper and Lower High Schools, Berkeley, Calif. Pages xvii+322. 14×20 cm. 1915. Charles Scribner's Sons, New York.

The distinctive feature of this book is the use of applied problems to develop the concepts of algebra. This part of the work, well planned and based on material within the experience of high school pupils, together with emphasis on equations, union with geometry, and the use of inductive methods, makes a textbook which will enable pupils to understand the use and value of algebra. The symbolism and operations of algebra grow out of real arithmetical problems, in many of which the data are secured by the pupil. Grades of roadbeds, roofs of houses, levers, belted pulleys in sewing machines, bolts and nuts, gears, bicycle pumps, and so on, furnish good problems for the development of algebraic concepts. With a clear understanding of literal numbers gained through these problems, the pupil is ready to attack with some interest the purely formal and abstract side of his algebraical work. This book can be heartily commended to the attention of secondary school teachers, since it evidences the accomplishment of the author's purpose to write a book "not 'for algebra's sake' but 'for the pupil's sake.'" H. E. C.

Methods in Plant Histology by Charles J. Chamberlain. Pages xi+314. 15½×22 cm. Third revised edition. 1915. $2.25 net. University of Chicago Press.

This valuable work, which has been the standard handbook of microtechnique in this country since its publication fourteen years ago, has been thoroughly revised and brought up to date. While the general arrangement of materials is unchanged, a large part of the book has been rewritten in accordance with the latest developments in methods. A chapter has been added in which the applications of photography to microscopic study are discussed.

The general excellence of the work is so generally recognized that detailed discussion is superfluous. W. L. E.

Applied and Economic Botany, by H. Kraemer. Pages vi+806., 424 figures. 15×23 cm. $5.00. Published by the author. For sale by M. G. Smith, 145 N. 10th St., Philadelphia. 1914.

The author of this book is a member of the Committee for Revision of the Pharmacopœia of the United States, and an authority in his own field of pharmacognosy. He has brought together in this book a very large volume of information about plants and their uses. Naturally, the economic facts presented are pharmacological rather than agricultural. For precisely that reason the book has the greater interest to teachers of botany. The agricultural implications of botany have been brought to our attention of late in numerous textbooks, but the multitude of facts regarding botanical food materials and drugs that one wishes to know have not heretofore been available in the literature with which teachers of botany are likely to be familiar.

Kramer discusses such widely variant questions as algæ and water supplies, poisonous and edible fungi, the popping of corn, the nature of ivy poison, and the serum treatment of hay fever.

The chapters which will probably be of most interest to secondary school teachers are those on "Cell-contents and Forms of Cells,"

"Angiosperms Yielding Economic Products," and "Cultivation of Medicinal Plants." The latter chapter is particularly interesting and though brief, constitutes possibly the best available summary of the matter now available. There is also a chapter on botanical nomenclature which includes a thirty-page glossary of generic and specific names with their derivation.

The use of the book as a reference work is facilitated by a splendid index which gives approximately six thousand citations. W. L. E.

A Manual of Weeds, by Ada E. Georgia. Pages xi+593. 12.5×19 cm. 1914. The Macmillan Co., New York.

In a manual of any group of plants, one expects to find keys and other means of identification. This book has no key of any sort. Since the species treated are arranged in the accepted botanical order, it is possible to find a given description by looking for it in its appropriate place in the sequence, or by consulting the index. Either method presupposes that the name of the plant is known.

The treatment of each species includes common and scientific names, data regarding time of flowering and seeding, range and habitat, simplified botanical description, and a discussion of means of control. Most of the species are figured. Counting an average of one species to a page, there are probably nearly six hundred species treated. The range covered is the whole of the United States and Canada.

The author has brought together a body of important information not before readily available. This is made readily available, with the reservation noted above, by means of an index of over two thousand entries.

W. L. E.

Plant Breeding, by L. H. Bailey, New Edition, revised by Arthur W. Gilbert. Pages xix+474. 13×19 cm. 1915. $2.00. The Macmillan Co., New York.

The present is the fifth edition of this well-known work since its first publication twenty years ago. It has undergone considerable revision at the hands of Dr. Gilbert, who has attempted to incorporate the advances in the knowledge of the subject which have been made in the nine years which have elapsed since the publication of the fourth edition.

The book retains the effective style which characterizes the works of Professor Bailey and is therefore suitable for use as a popular treatise, as an elementary textbook, or for general library use. It may be expected to continue the popularity enjoyed by the earlier editions.

W. L. E.

The Elements of Physical Chemistry, by Harry C. Jones, Professor of Physical Chemistry in the Johns Hopkins University. Fourth Edition, revised and enlarged. Pages 672. 15×22×4 cm. Illustrated. Cloth. 1915. $4.00. The Macmillan Co., New York.

This new edition of a well-known text is intended primarily for college students in the later years of the college course and for graduate students in the university. It might well, however, find a place in the chemical library of every secondary school that can afford more than the necessary reference books, for the conceptions of modern physical chemistry should be familiar to the secondary teacher, even though some of them may not yet be brought down to the comprehension of his classes.

As an example of the sort of assistance that the secondary chemistry teacher may get from this volume, we would cite such topics as the nature of colloids, the theory of indicators, the more recent modifications of the ionic theory (for example, the part played by the few ions furnished

by water itself—in the electrolysis of dilute sulphuric acid), the nature of solid solutions, the several methods of determining molecular weights and recent improvements in their use, the theory of solutions, etc.

While some parts of the work presuppose considerable chemistry and some use of the calculus—very much may be gained from the use of the book as a reference—even by those teachers who are not thus equipped.

F. B. W.

Laboratory Experiments on Food Products, Designed Especially for Use with Bailey's "The Source, Chemistry and Use of Food Products," by E. H. S. Bailey, Ph.D., Professor of Chemistry and Director of Chemical Laboratories, University of Kansas. Pages vi+44. 13.5×20.3 cm. Paper. 1915. Price, 25 cents. P. Blakiston's Son & Co., Philadelphia.

This little manual contains a large number of tests for food materials as well as for common adulterants. While intended for the use of pupils who have had at least a year of general chemistry, the manual should be of value also to those teachers of first year chemistry who are trying to make their work apply as far as possible to the things of the home. Students of domestic science should find the little book very useful. The early experiments show how to detect water, carbon, nitrogen, organic acids, and mineral salts in foods. Next, cellulose, starch, dextrine, gluten and gliadin are tested for.

The hydrolysis of starch by the action of dilute acids and by enzymes is next shown.

Alum and copper sulphate are tested for in flours. Baking powders are tested. The sugars are then studied, then alcoholic beverages, vegetables, legumes and fruits.

Next many foods are tested for adulterants or preservatives.

Cheese, eggs, spices, condiments, and nonintoxicating beverages complete the list of topics.

The directions are clear although rather brief and the tests generally well chosen.

F. B. W.

Laboratory Exercises, Arranged to Accompany "First Course in Chemistry," McPherson and Henderson, Ohio State University. Pages ix+128. 13.5×19×1 cm. Drawings. Cloth. 1915. 40 cents (in biflex binder, 60 cents). Ginn & Co.

The earlier part of this manual deals with the necessary manipulation of apparatus and with the usual list of stock experiments, such as the preparation of hydrogen and oxygen, determination of the weight of a liter of oxygen and the percentage of oxygen in potassium chlorate, the determination of hydrogen equivalents, etc. Solutions and solubility are then studied. Acids, bases and salts and titration are next considered. The preparation of ammonia and nitric acid and of the commoner oxides of nitrogen follow. Hydrogen sulphide, sulphuric acid, hydrogen fluoride and chlorine are prepared. Then hydrochloric acid, bromine and hydrobromic acid, iodine and hydro-iodic acid come in order. Compounds of carbon are next studied and here some simple food tests are given. The remaining thirty-five exercises deal partly with the metals and tests for them and partly with so-called applied chemistry, such as the removal of stains, analysis of baking powders, mordants in dyeing, dyes in foods, etc.

The directions are clear and concise. Necessary precautions are always given. Leading questions help the pupil to focus attention on essentials. A sufficient number of exercises is given so that the teacher may choose a course suited to his time limitations.

F. B. W.

SOME DEPOSITS OF MICA IN THE UNITED STATES.

Mica mining in the United States began with the opening of the Ruggles mine, in Grafton County, N. H., about 1803. Later, other mines were opened in New Hampshire, which for many years furnished practically all the mica in this country. A report by Douglas B. Sterrett, recently issued by the United States Geological Survey as *Bulletin 580-F*, describes the Ruggles mine as well as other mica deposits in the United States.

Mica is a valuable mineral in the industrial world, where it meets a demand not supplied by any other material, so that the sources of supply are of both present and future importance. Many good mica deposits are known in the United States and the production is increasing. The imports of mica are generally greater than the domestic production, but the mica mines of this country could be made to supply all but that small part of the domestic demand which calls for the softer Canadian "amber" mica. Good mica mines have been worked in North Carolina, New Hampshire, South Dakota, Idaho, New Mexico, Virginia, South Carolina and Alabama, and promising deposits are known in several other states.

The occurrence of mica deposits in many countries insures future supplies of mica for the world for some years to come, and the numerous undeveloped mica deposits of the United States may be considered among these resources. Under present conditions the mica deposits of the United States will probably continue to yield a considerable part of the mica used in this country.

A copy of the report may be obtained free on application to the Director of the United States Geological Survey, Washington, D. C.

WHAT IS A PLACER?

A placer is an unconsolidated deposit accumulated by mechanical processes, carrying one or more minerals in commercial quantities. All placers are secondary deposits—that is, the material of which they are composed was originally derived by erosion of bed rock. Although it is undoubtedly true that under certain conditions nuggets of placer gold have been enlarged through chemical precipitation, yet this action is a negligible quantity in placers. Placers may be derived solely by rock weathering without water sorting, but more commonly are the result of water transportation, sorting and deposition. Many of the richest placers are those formed by the erosion of older placers and the reconcentration of their gold.